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THE

PHYSICAL REVIEW.

ON THE DISTRIBUTION OF ENERGY IN THE VISIBLE SPECTRUM OF AN ACETYLENE FLAME.

By E. P. Hyde, W. E. Forsythe and F. E. Cady.

Synopsis.

A knowledge of the distribution of energy in the visible spectrum of an acetylene flame has become important within the last few years through the use of this flame, in cylindrical form, in investigations of the visibility of radiation. It can be shown by computation that the data on acetylene published by Coblentz form a curve in the visible spectrum which will not agree with that of a black body at any temperature to better than 7 or 8 per cent. As this would mean that no color match could be obtained and as previous experience of the authors had led to the conclusion that the energy curve of acetylene differed in shape from that of a black body only in the extreme red, a short investigation was undertaken to verify this conclusion.

Tungsten lamps whose current color-temperature relation was carefully determined in this Laboratory, were sent to the Eastman Kodak Company and to the Bureau of Standards with the request that they be compared with the acetylene flame and the current for color match be found. The results gave an average value of 2360° K. ± 10° K., and neither laboratory reported any difficulty in obtaining a match in color. However the Bureau of Standards reported a difference amounting to about 75° K. between the flame as given by the Eastman standard burner and that given by the "Crescent *Aero*" burner, the latter being higher.

The spectral distribution of the flame was measured by means of a spectrophotometer and a spectral pyrometer and the results gave a curve agreeing within the limits of error with that of a black body at 2360° K. In the extreme red, beyond 0.70 μ there was indication of a higher emissivity for the acetylene. A photographic method gave results corroborating those just mentioned.

A test of the sensibility of the color-match method to show differences in the spectral energy curve, showed that if two spectral curves matched at 0.5 μ and 0.7 μ and differed by as little as 4 per cent. in the middle of the spectrum, the two light sources could not be made to match in color.

In conclusion it is recommended that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval from 0.4 μ to 0.7 μ should be taken as identical with those of a black body at 2360° K.

THE acetylene flame has frequently been employed in spectroradiometric work, and it has been subjected to investigation with respect to the distribution of energy in its spectrum as well as with respect to its constancy and reproducibility as a standard of luminous intensity. At the present time it demands a special consideration in view of its employment as a standard source in the determination of the visibility of radiation. In the recent investigations of the spectral energy distribution of acetylene a cylindrical flame has been used,—either as produced by a single-jet "Crescent *Aero*" burner as employed by Coblentz,[1] or as given in the standard lamp recommended by the Eastman Kodak Company laboratory, consisting of a 1/4-foot "Bray" tip and a limiting diaphragm restricting the utilizable portion of the flame to 3 mm., as thoroughly described and discussed by Jones.[2] The present note deals exclusively with the cylindrical flame, and the data reported are therefore comparable only with other data obtained with a flame of this form.

In connection with a general survey of the various published investigations of the visibility of radiation, including a careful study of the methods and apparatus employed, the authors observed that the data given by Coblentz for the spectral energy distribution of acetylene showed that the spectral energy curve departed somewhat from that of a black body, indicating that no temperature of a black body could be found for which the distribution of energy in the visible spectrum would be sensibly the same as that of the acetylene flame. This discrepancy is most pronounced in comparing the energy distribution of the two sources in the extreme blue end of the spectrum, when the black-body energy distribution is computed from Wien's equation for a temperature such that at 0.48 μ and at 0.68 μ the emission intensities are relatively the same for the two sources. This comparison is shown in Table I., column 2, where are given the ratios of the emission intensities of a black body at 2434° K. to the published values for acetylene referred to above, the ratios at 0.48 μ and at 0.68 μ arbitrarily being taken as unity. An inspection of this column shows that the relative emission intensity of acetylene at 0.40 μ is approximately 1.6 times that of the black body at 2434° K., and although the deviations in the middle of the spectrum are much smaller,—of a different magnitude,—they would still seem sufficiently large to be indicated by a difference in hue if an attempt were made to match the acetylene and the black body in color, as in the determination of the color temperature of acetylene. It therefore occurred to the authors to obtain the color temperature of the acetylene flame in order

[1] Bul. Bur. of Standards, 13, p. 355, 1916–17.

[2] Trans. Ill. Eng. Soc., 9, p. 716, 1914.

to find: (1) Whether any appreciable color difference between the black body and the acetylene flame would be observed when the former was set at the nearest temperature to match the acetylene in color; and (2) whether the color temperature found would indicate a distribution of energy consistent in some region of the spectrum with that given by Coblentz.

Since a standard acetylene burner was not available in Nela Research Laboratory, two carefully prepared tungsten lamps for which the energy distributions at different voltages were known by comparison with a black body, were sent respectively to Dr. Jones, of the Eastman Kodak Company and to the Bureau of Standards, with the request that the voltage for color match with the standard acetylene flame be determined. No mention of the possible difficulty of obtaining a satisfactory color match was made, the hope being that some statement to this effect would be forthcoming if difficulty were experienced. Previous observations by the authors employing acetylene flames of types other than the standard had shown no such marked selectivity, and consequently it was not expected that such a difficulty would be encountered in the present case.

The two laboratories very kindly made the requested determinations and reported the voltages found for color match. According to the color scale in use in Nela Research Laboratory,[1] these two voltage determinations conduced to the following color temperatures of the standard acetylene flame:

Lamp sent to Bureau of Standards—Voltage 112.3, Color Temp. 2434° K.
Lamp sent to Eastman Kodak Co.—Voltage 102.3, Color Temp. 2352° K.

The difference between these two determinations was so great that it seemed advisable to procure a standard Eastman burner and undertake in Nela Research Laboratory an independent determination of its color temperature. The result of this investigation was a value of 2360° K.± 10° K. when the lamp was operated at a pressure of 9 cm. and fed with acetylene from a Thorn and Hoddle generator, purified by passage through "Curaze," a material furnished with the generator. A study of the effect of pressure indicated that only very small changes (a few degrees) in color temperature resulted from a change in pressure from 9 cm. to 7.5 cm.,—the pressure employed by Coblentz in his "Crescent *Aero*" burner. Measurements were also made with acetylene obtained from a Prestolite cylinder and passed through a solution of bisulphite of soda to get rid of possible acetone. This experiment gave a somewhat higher

[1] These temperatures are based upon Wien's equation with c_2 taken as 14350 μ deg. and upon the temperature of the gold point taken as 1336° K. On this scale the palladium point has been found to be 1828° K. For convenience, a black body held at this temperature is used in calibrating the pyrometers.

color temperature with a maximum difference of 30°. Owing to uncertainty as to the purity of the gas in the tank these values were given no weight.

During the investigation it was found that for a time after the burner was lighted, when supplied with acetylene from a fresh charge of the Thorn and Hoddle generator, the color temperature was much higher than normal. This may be due to an admixture of air with the acetylene. By permitting the generated acetylene to escape for a while before connecting the supply tube to the burner the effect was eliminated.

In addition to the determinations of color temperature, measurements of the spectral energy distribution were also made with both the spectrophotometer and the spectral pyrometer. With the former instrument readings were made over the range 0.48 μ to 0.68 μ, the results indicating a black-body distribution corresponding to the temperature of 2360° K., the same as found by direct color match. The measurements with the spectral pyrometer extended from 0.45 μ to 0.76 μ and also confirmed the previous results in the region from 0.47 μ to 0.70 μ. The uncertainty of the measurement at 0.45 μ was so great that the observed deviation of 4 per cent. is within the experimental error. Beyond 0.70 μ the measurements indicated that the acetylene flame had a slightly greater emissivity (amounting to about 7 per cent. at $\lambda = 0.75\ \mu$) than that of a black body at 2360° K., consistent with results obtained previously by one of the present authors.[1] Owing to the difficulties of measurement at the extreme wave-lengths too much weight should not be assigned to the observed values in this region; however, all measurements showed the increased emissivity in the red.

It is seen that all the measurements,—those of direct color match and those of spectral energy distribution, conduce to the value 2360° K. as the color temperature of the standard Eastman burner. This result is consistent within experimental errors with the value 2352° K. obtained through the incandescent lamp sent to the Eastman laboratory and compared with a standard burner there through the courtesy of Dr. Jones. It differs, however, by approximately 75° from the value obtained from the incandescent lamp which was matched in color with the "Crescent *Aero*" burner at the Bureau of Standards through the kind collaboration of Messrs. Crittenden and Coblentz.

Another determination through an incandescent lamp was therefore requested of the Bureau of Standards, with a transmitted statement of the results given above. The Bureau courteously undertook a second determination, this time making measurements on both the "Crescent

[1] Jour. of Frank. Inst., 170, p. 30, 1910.

Aero" burner and on the Eastman standard burner for which Dr. Coblentz had reported[1] the same spectral energy distribution except for a slightly greater emissivity in the extreme red end of the spectrum beyond 0.68 μ. The results of this determination showed a color temperature of 2450° K. as compared with the value 2435° K. previously obtained for the "Crescent *Aero*" burner, and a color temperature of 2370° K. for the Eastman burner. The latter value agrees very well with that (2360° K.) obtained in this Laboratory, but the value found for the other burner, though fairly consistent with the previous determination, indicates clearly that the Eastman burner has an energy distribution distinctly different from that of the "Crescent *Aero*" burner. According to the measurements of Dr. Coblentz the only difference is to be found in the spectral region beyond 0.70 μ, but it is impossible to account for the difference in color temperature of 75° or 80° on this basis. It would also seem inadequate to account for the difference on any such grounds as the quality of acetylene gas used or the effect of atmospheric conditions, since the various determinations on the Eastman burner, made at different places and under different conditions, show a maximum variation of only 18°.

As already stated, the present study was suggested by the deviation of the published results of the energy distribution of the standard acetylene burner from that of a black body. By reference to Table I. in which are given the ratios of the emission intensities of both the "Crescent *Aero*" and the Eastman standard burners, as reported by Coblentz, to those of a black body at 2360° K. and at 2450° K., the ratio at 0.48 μ arbitrarily being taken as unity, it is seen that the reported energy distribution of neither acetylene burner corresponds to that of a black body at any temperature. Not only would it appear that acetylene has an extremely high emissivity in the region of short wave-lengths (0.40 μ to 0.45 μ) but also that in the middle region of the visible spectrum (0.5 μ to 0.7 μ) the deviations are sufficiently large to justify the expectation that no exact color match could be obtained with a black body at any temperature. One would expect, if the published data were accurate, that the acetylene flame would have a slightly purplish color in comparison with which the black body would appear slightly greenish by contrast. Neither of the other two laboratories has reported any such observed effect, and careful observation in Nela Research Laboratory with the phenomenon especially in mind has failed to detect such an effect.

The question naturally arises as to the sensibility of the color-match method. Long experience in this Laboratory has established the conclu-

[1] *Loc. cit.*

TABLE I.

Ratio of Emission Intensities of a Cylindrical Acetylene Flame as Found by Coblentz for the "Crescent Aero" and the "Eastman Standard" Burners, to those of a Black Body at Various Temperatures. (Ratio at 0.48 μ taken as unity in each case.)

Wave-length.	Ratios Black Body to "Crescent *Aero*" Burner.			Ratios Black Body to "Eastman Standard" Burner.	
	2434° K.	2360° K.	2450° K.	2360° K.	2450° K.
0.40 μ	0.62	0.58	0.62	0.58	0.62
.42	.74	.70	.75	.70	.75
.44	.86	.83	.87	.83	.87
.46	.94	.93	.95	.93	.95
.48	1.00	1.00	1.00	1.00	1.00
.50	1.04	1.06	1.04	1.06	1.04
.52	1.05	1.09	1.04	1.09	1.04
.54	1.05	1.10	1.04	1.10	1.04
.56	1.06	1.12	1.04	1.12	1.04
.58	1.05	1.12	1.04	1.12	1.04
.60	1.05	1.14	1.03	1.14	1.03
.62	1.04	1.14	1.02	1.14	1.02
.64	1.03	1.14	1.01	1.14	1.01
.66	1.01	1.12	.99	1.12	.99
.68	1.00	1.11	.97	1.11	.97
.70	.99	1.11	.96	1.10	.95
.72	.98	1.10	.95	1.09	.93
.74	.97	1.10	.93	1.08	.91

sion that a relative change in emission intensity at 0.5 μ and at 0.7 μ of 2 per cent., corresponding to a change of approximately 0.5 per cent. in the temperature of a black body in the region of 2400° K. is readily observable. But although this sensibility obtains in matching a black body with another source having approximately a black-body distribution, it tells nothing about the sensibility of the method in comparing with a black body a source which has a relatively high or relatively low emissivity in the middle of the visible spectrum. An important point to determine is the necessary magnitude of this departure from a black-body distribution in order that it should be observed in the color difference as seen in the photometer.

An experiment was arranged to establish this point. A piece of plate glass was selected which by absorption changed the color of the light from a tungsten lamp to a slightly greenish hue. The change in color was so small that close observation was required to detect it. That it was appreciable was indicated, however, by the fact that out of eleven trials an observer judged correctly ten times whether the glass was or was not interposed, and the condition of the experiment was such that no other indication than the greenish tint due to the absorption of the glass could

be used as a basis of judgment. Having shown that the color difference between the direct light from the lamp and that transmitted by the glass was appreciable a determination of the spectral transmission of the glass was undertaken. This determination was made using both a spectro-

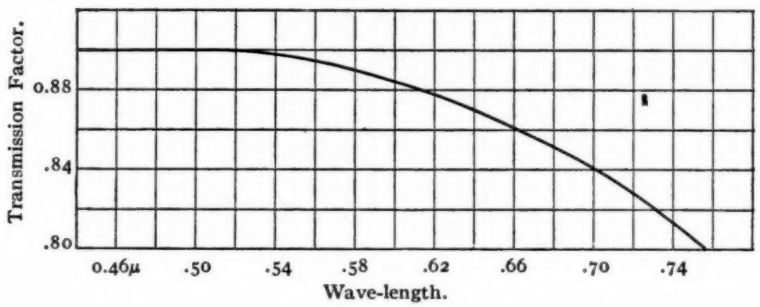

Fig. 1.

Transmission of a Selected Piece of Plate Glass of Slightly Greenish Hue.

photometer and a spectral pyrometer, the two methods giving relative transmission curves differing by not more than 1 per cent.

The resultant transmission curve for the glass is shown in Fig. 1. It is seen that the glass is slightly bluish as well as having a relatively larger transmission in the green, and consequently the nearest color temperature of light transmitted through the glass would be higher than that of the direct light from the lamp. For the source employed these two color temperatures were found by experiment to be 2414° K. and 2374° K. respectively. If now the computed emission intensities of a black body at 2374° K. are multiplied by the relative transmission factors given in Fig. 1, and the resultant values compared with the computed emission intensities of a black body at 2414° K., the magnitude of the excess in green of the light transmitted by the glass is shown. The results of these computations are given in Table II., computed for every 0.04 μ. It is seen that the relative difference between the two energy distributions which were observed by the color-match method to be appreciably different, is at most 3.8 per cent. between the wave-length limits of 0.48 μ and 0.72 μ, so that it may safely be concluded that corresponding relative differences as large as 6 or 7 per cent., such as would be indicated by comparison of the measurements on acetylene by Dr. Coblentz with the energy distribution of a black body at the nearest color temperature and over the same wave-length interval, should easily be observed.

As stated in a previous paragraph the largest deviation between the published emission intensities of acetylene and the computed emission

intensities of a black body at color match (2360° K. for the Eastman standard burner and approximately 2440° K., the average of the two values given by the Bureau of Standards, for the burner used by Coblentz)

TABLE II.

Modification of Black-Body Radiation by Transmission through a Piece of Slightly Selective Plate Glass.

Wave-length.	Transmission Factor of Glass.	(*a*) Relative Emission Intensities of Black Body at 2374° K. as Modified by Glass.	(*b*) Relative Emission Intensities of Black Body at 2414° K.	Column (*a*) — Column (*b*) in Per Cent.
0.48 μ	0.90	120	120	±0.0
.52	.90	212	208_5	+1.7
.56	$.89_5$	333	326	+2.1
.60	$.88_5$	480	468	+2.6
.64	.87	641	630	+1.7
.68	.85	806	804	+0.2
.72	.83	969	981	−1.2

is to be found in the blue end of the spectrum. The authors' measurements with spectrophotometer and spectral pyrometer could be extended in this region only to 0.45–0.48 μ, so that some other method was required. For this purpose photography was employed, and although its indications as used are not accepted as conducing to accurate quantitative results, they may be depended upon for approximate values.

The method employed consisted in taking a spectrogram of the acetylene flame, and then on an adjacent portion of the plate, a spectrogram with the same exposure time of a standard tungsten lamp operating at the voltage corresponding to the color temperature (2360° K.) of the acetylene burner. From previous measurements of the brightness temperatures of the two sources, it was known to be necessary to interpose a 45° rotating sectored disk in the path of the light from the tungsten lamp in order to bring the two spectra to approximately the same intensities at 0.55 μ. The two spectrograms thus obtained were measured for photographic density at the two wave-lengths 0.55 μ and 0.41 μ, the method of measurement consisting in determining the transmission (corrected for the plate) of the two spectrograms at the two wave-lengths by the use of a pyrometer. A pyrometer with a very small filament was found to be a most convenient instrument for measurements of this kind.

The results of this experiment showed roughly the same relative emission intensities of the Eastman standard acetylene flame and a black body at 2360° K. between the two wave-lengths,—0.55 μ, in the central region of the visible spectrum, and 0.41 μ, well out in the extreme

blue end of the spectrum. If Coblentz's values for acetylene were accepted the emission intensity at 0.41 μ for acetylene should be about 75 per cent. more than that of the black body at 2360° K., as compared with the same emission intensities at 0.55 μ, and so the photographic density of the spectrogram for acetylene should be relatively much greater in the short wave-length region.

As a check experiment a series of spectrograms were taken as before, except that the 45° sectored disk was replaced by a 60° disk. For the same exposure time this would expose the plate to 60/45 or 4/3 the quantity of light for the tungsten source, and would show whether an increase of 33-1/3 per cent. in the luminous flux would produce a very appreciable difference. The result was without question the photographic density of the tungsten spectrograms being uniformly greater throughout the spectrum, and yet the difference in exposure for the two spectrograms was less than one half that which would have resulted between the two chosen wave-lengths of the acetylene spectrogram if the relative emission intensities of acetylene were of the order of magnitude found by Coblentz.

It should be noted that the color temperature of acetylene has been assumed to be 2360° K., or that of the Eastman burner, rather than 2440°, that found at the Bureau of Standards, for the burner employed by Coblentz in his principal measurements. But it should also be noted that he found the same emission intensities for the Eastman burner throughout the visible spectrum except in the red beyond 0.68 μ. Moreover, the results would suffer no change in magnitude if the higher color temperature had been assumed.

From these various experiments it must be concluded that the emission intensities published by Coblentz for a cylindrical acetylene flame of the type found in the Eastman standard burner are subject to considerable uncertainty. In the central portion of the visible spectrum the results would seem to be uncertain to the extent of at least 5 per cent., and in the blue end of the spectrum the values given are too high by many per cent. At 0.41 μ for example, the published value would seem to be too high by at least 50 per cent.

In connection with the above experiments measurements were made of the variation of the brightness temperature and of the color temperature for the different parts of the flame seen through the 3 mm. diaphragm used with the Eastman standard burner. There was found a variation of about 25° in the brightness temperature and somewhat more in the color temperature between the top and bottom of the flame. The brightness temperature of the flame at the middle of the diaphragm was found to be 1728° K. at 0.665 μ, which agrees well with the average brightness temperature of the entire portion of the flame exposed by the diaphragm.

Until more accurate data are available the authors would recommend that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval 0.4 μ to 0.7 μ, should be taken as identical with those of a black body at 2360° K. As stated in the previous paragraph, the brightness temperature at 0.665 μ was found to be 1728° K. From these two sets of data a table of emission intensities may readily be computed.

NELA RESEARCH LABORATORY,
CLEVELAND, OHIO,
May, 1919.

When the manuscript of this paper was sent to the printer, a copy was forwarded to Dr. Coblentz who later informed the authors that the "Crescent *Aero*" burner used in the color-matching tests at the Bureau of Standards, was not the same as the one used in his work on visibility. The burner used in the visibility work had an opening of 8 mm., whereas that used in the color-match had a 10 mm. opening. He also stated as the result of a correction in the blue end of the spectrum, his revised data give a distribution agreeing with that of a black body at 2360°K to within 3% from 0.50μ to 0.74μ.

ON THE SPECTRUM OF X-RAYS FROM AN ALUMINIUM TARGET.[1]

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SYNOPSIS.

Aluminium Spectrum.—The object of the research has been to investigate the question whether or not the frequencies of the K series are the highest X-rays frequencies characteristic of a chemical element. Several investigators have found experimental evidence, which they interpret as indicating the existence of characteristic emission or absorption of frequency higher than those in the K series. For aluminium the wave-lengths corresponding to these frequencies are stated to be $.37 \times 10^{-8}$ cm. and $.49 \times 10^{-8}$ cm. The authors of this paper have examined the emission spectrum of aluminium between the wave-lengths $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. Four small peaks appear on the curves, indicating characteristic radiation at wave-lengths $.622 \times 10^{-8}$ cm., $.705 \times 10^{-8}$ cm., $.975 \times 10^{-8}$ cm. and 1.18×10^{-8} cm. The first two belong to the K series of molybdenum, and undoubtedly come from the metallic molybdenum in the Coolidge carthode in the X-ray tube. The last two belong to the L series of lead, and undoubtedly come from the lead screens containing the slits through which the X-rays passed before they reached the X-ray spectrometer.

No other peaks appear on the curve, and this leads to the conclusion that, within the range examined, aluminium has no emission lines the intensities of which amount to as much as 2 per cent. of the general radiation in the neighborhood.

SEVERAL investigators[2] have obtained experimental evidence, which they interpreted as indicating the existence of the emission or the absorption of X-radiation characteristic of a chemical element (the J-series), and of higher frequency than those in its K-series. Barkla and white (*l.c.*) measured the coefficient of absorption of X-rays in copper, aluminium, paper, water, and paraffin-wax. On platting these coefficients against the wave-lengths, and also on platting the coefficients for one substance against those for another (copper) certain breaks in the curves appeared, similar to, but very much smaller than the breaks that occur at the critical absorption wave-lengths associated with the K and L series of characteristic X-rays. From these breaks they inferred the existence of X-radiation characteristic of aluminium, oxygen and carbon at the wave-lengths $.37 \times 10^{-8}$ cm., 39×10^{-8} cm. and $.42 \times 10^{-8}$

[1] A paper read at the New York meeting of the American Physical Society, March 1, 1919.

[2] C. G. Barkla, Roy. Soc. Phil. Trans., 217, pp. 315–360, Aug. 29, 1917. C. G. Barkla and Margaret P. White, Phil. Mag., 34, pp. 270–285, Oct., 1917. C. M. Williams, Roy. Soc. Proc., 94, pp. 567–575, Aug. 1, 1918.

cm. respectively. In these experiments the X-rays were not homogeneous, and the wave-lengths were not measured by means of an X-ray spectrometer. They were calculated from the relation between the wave-length and the mass coefficient of absorption in aluminium. At a meeting of the American Physical Society in December, 1914, one of us proposed the equation

$$\frac{\mu}{\rho} = 14.9 \times \lambda^3 \text{ (}\lambda \text{ in ångströms)}$$

for this purpose, and it has been used to measure the wave-lengths of the general radiation.[1] It gives the effective wave-length, which may be defined as the wave-length of the homogeneous radiation that would have the same coefficient of absorption as the general radiation in some standard substance.

Williams (*l.c.*) measured the coefficient of absorption of *homogeneous* X-rays in aluminium and in copper. The homogeneous beam of rays was produced and its wave-length measured by means of an X-ray spectrometer. He took the precaution of so regulating the voltage applied to the X-ray tube that no X-rays in spectra of higher order than the first should be reflected by the crystal at the grazing angles used.

He found a small break in the coefficient of absorption wave-length graph for aluminium at $\lambda = .49 \times 10^{-8}$ cm., which is 30 per cent. longer than the wave-length determined by Barkla and White.

Williams also found a break in the corresponding graph for copper, but the break is in a direction opposite to that occurring at the characteristic absorption wave-lengths in the K and L series.

On account of the great importance in the theories of radiation and the structure of matter of knowing exactly what the highest X-ray frequency characteristic of a chemical element really is we decided to investigate the spectrum of the X-rays *emitted* by an aluminium target.

The Wave-lengths of the X-rays we have examined lie between $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. This is the portion of the spectrum usually obtained from an ordinary X-ray tube excited by a difference of potential of 71,200 volts, and includes with ample margins the region in which the above mentioned scientists found the breaks in the absorption curves, and in which, therefore, the J characteristic radiation is supposed to lie.

As the shortest wave-length in the K series of aluminium is 7.98×10^{-8} cm. the range of frequencies in our experiments extends from 6.34 to 43.8 times the highest frequency in this K series.

[1] Duane & Hunt, PHYS. REV., Vol. 6, No. 2, p. 169, Aug., 1915.

To produce X-rays of higher frequency than the upper limit in our experiments requires a difference of potential greater than 71,200 volts. At this difference of potential the X-ray tube began to show signs of weakening, and we did not push the research further for fear of destroying it.

The thickness of the glass walls of the X-ray tube fixes the lower limit of frequency. X-rays longer than about 1.26×10^{-8} cm. are almost completely absorbed by the glass.

In making the measurements we used the apparatus described in the PHYSICAL REVIEW for December, 1917, p. 624. This consisted of an X-ray spectrometer with a calcite crystal, the beam of X-rays being defined by two slits in lead blocks, which lay between the X-ray tube and the spectrometer. The electric current through the tube came from

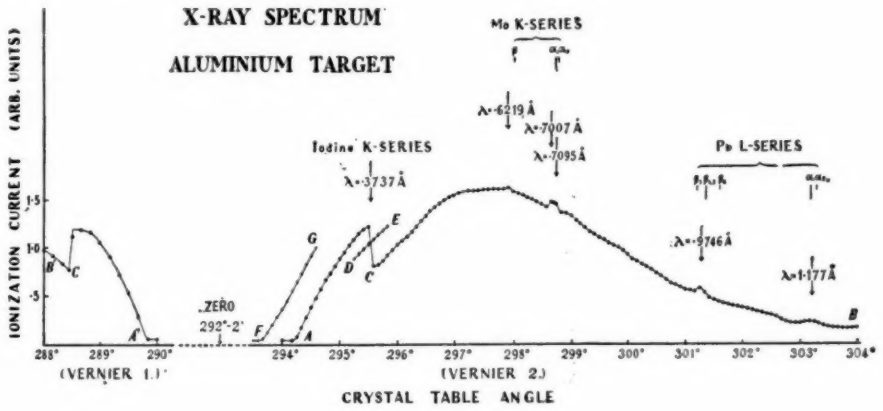

Fig. 1.

a high tension transformer connected to a system of kenotrons and condensers, which produced approximately a constant voltage.

The X-ray tube (kindly sent us by Dr. W. D. Coolidge) was similar to the ordinary Coolidge tube, except that an aluminium target replaced the usual tungsten target. The cathode in these tubes consists of a hot tungsten wire coiled in the form of a circular disk and surrounded by a shield of metallic molybdenum.

The figure contains the curves obtained by platting the currents in the spectrometer's ionization chamber against the readings of the verniers attached to crystal table.

We took readings at intervals of 5 minutes of arc, having purposely regulated the breadths of the slits so as to eliminate the possibility that a peak representing characteristic radiation might lie between two such readings.

Curves AB and $A'B'$ represent an experiment in which the difference of potential applied to the tube amounted to about 53,200 volts. The points A and A', where the curves begin to rise above the natural ionization leak of the instrument, correspond to the frequency which, when multiplied by Planck's action constant, h, equals the energy acquired by an electron in the X-ray tube as given by the product of the difference of potential into its electric charge.[1]

The position of the zero on the scale was estimated from these two points A and A'. A correction of 45″ of arc must be subtracted from the double grazing angle of incidence 2θ on account of the scale's eccentricity, when calculating the wave-lengths from the usual formula $\lambda = 6.056 \times \sin\theta \times 10^{-8}$ cm.

At the points marked C and C' occur sharp breaks in the curves. These are due to the fact that the ionization chamber contained methyl iodide. The critical ionization wave-length of iodine is $.3737 \times 10^{-8}$ cm.,[2] and X-rays of shorter wave-length than this ionize the gas much more strongly than X-rays of longer wave-length do.

As the break in the absorption curve obtained by Barkla and White (*l.c.*), from which they inferred the existence of characteristic J radiation occurs at wave-length $.37 \times 10^{-8}$ cm., and as the break in our curve at C might completely mask a peak representing characteristic emission lines, we repeated our measurements over wave-lengths in this neighborhood using ethyl bromide in the ionization chamber instead of methyl iodide. The curve DE represents this experiment, and no peak appears in it that can be interpreted as indicating characteristic emission lines.

Further no peak appears on the curve AB in the neighborhood of wave-length $.49 \times 10^{-8}$ cm. at which Williams (*l.c.*) found a break in the absorption curve.

There are, however, two small peaks at wave-lengths $.6219 \times 10^{-8}$ cm. and $.7051 \times 10^{-8}$ cm. Within the limits of experimental error these are the wave-lengths of the β and α lines in the K series of molybdenum, and it is reasonable to suppose that these peaks are due to the presence of the metallic molybdenum in the Coolidge cathode. It is uncertain whether they are to be ascribed to primary radiation from a thin film of molybdenum deposited on the target during the excessive heating process to which these X-ray tubes are subjected during their exhaustion, or to tertiary, etc., rays excited in the target by secondary rays from the molybdenum in the cathode.

At wave-lengths $.9746 \times 10^{-8}$ cm. and 1.177×10^{-8} cm. appear two

[1] Duane & Hunt, PHYS. REV., August, 1915, p. 169.

[2] Duane and Hu, PHYS. REV., June, 1918, p. 491.

more small peaks indicating characteristic radiation. These peaks correspond exactly with the β and α lines in the L series of lead, and, doubtless, are due to secondary rays from the lead blocks containing the slits through which the X-rays passed before they reached the spectrometer.

In order to extend the research to X-rays of shorter wave-length than those produced by a difference of potential of 53,200 volts, we took a series of readings with a difference of potential of 71,200 volts. The curve *FG* represents these measurements, and it contains no peak.

Molybdenum and lead are much more efficient radiators of X-rays than aluminium is, for their atomic numbers are higher, 47 and 82 respectively, instead of 13 for aluminium. Further, if the voltage applied to the X-ray tube lies considerably above that required to produce the characteristic rays of its target (as is the case with molybdenum and lead in our experiments), the characteristic radiation is many times more intense than the general radiation in its neighborhood. It is not surprising, therefore, that secondary or tertiary characteristic radiation from molybdenum and lead can produce effects amounting to a few per cent. of the general primary radiation from aluminium, upon which it is superposed, as indicated by our curves.

No peaks representing the emission of characteristic X-rays appear on our curves other than those corresponding to the K series of molybdenum and the L series of lead, and we conclude, therefore, that aluminium has no characteristic lines in its emission spectrum, between the wave-lengths $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. that amount to as much as 2 per cent. of the general radiation in the neighborhood and that can be produced by the voltages we employed.

HARVARD UNIVERSITY.

STUDIES OF THE ADSORPTION OF GASES BY CHARCOAL. II.[1]

BY HARVEY B. LEMON AND KATHRYN BLODGETT.

SYNOPSIS.

Problem.—When an activated and previously evacuated specimen of cocoanut shell charcoal is exposed to an excess of a mixture of oxygen and nitrogen, the two gases are unequally adsorbed. It is desired to determine how the equilibrium pressure at constant volume depends upon the composition of the mixture; the ratio of the mass of gas to the mass of charcoal being the same in all cases.

Results.—(1) There is a nearly linear relation between the logarithm of the equilibrium pressure and the per cent. of oxygen in the mixture. (2) The two gases in the mixture are not adsorbed independently. (3) A given volume either of a pure gas or of a mixture is less completely adsorbed if admitted at one time than if it is admitted in two portions, the second half being admitted after the adsorption of the first has reached equilibrium.

RELATIVE ADSORPTION OF MIXTURES OF OXYGEN AND NITROGEN. EQUILIBRIUM EFFECTS.

THE close connection between the boiling temperature of a gas and the amount which is adsorbed in charcoal at a low temperature has been recognized for many years. In 1907 H. Baerwald wrote:[2] "The more easily a gas is condensed, the more abundantly, then, it is adsorbed, the harder it is to expel it again from the charcoal, and the longer the process of adsorption lasts"; a statement which has been found true, in general, for all gases except hydrogen, whose adsorption is far greater than would be expected from its low boiling point.[3] Oxygen and nitrogen consistently follow this rule. Oxygen which boils at $-$ 182.7° C. is adsorbed and expelled so much more readily than nitrogen which boils at $-$ 195° C., that Dewar, after finding that the gas adsorbed by charcoal from air was 57 per cent. oxygen, recommended "that one of the most rapid means of extracting a high percentage of oxygen from atmospheric air is to adsorb it in charcoal at low temperatures and then

[1] This article is published with the approval of Major General William L. Sibert, Director of Chemical Warfare Service, U. S. A.

[2] H. Baerwald, "Ueber die Adsorption von Gasen durch Holzkohle bei tiefen Temperaturen," Ann. d. Physik, 23, 90–91, 1907.

[3] M. Georges Claude, "Sur l'adsorption des gaz par le charbon auz basses temperatures," Comptes Rendus, 158, 863, 1914.

to expel it either rapidly or slowly by heating the mass of charcoal to the ordinary temperature." [1]

The object of the present experiment was to investigate the adsorption of *mixtures* of oxygen and nitrogen, to learn if the adsorption increased continuously from that for pure nitrogen to that for pure oxygen as the percentage of oxygen present in the mixtures was continuously increased. It was also found profitable to investigate the behavior of the constituents when adsorbed separately, in order to determine whether or not they acted independently when adsorbed together as a mixture.

The apparatus was built in such a way that a few grams of charcoal could be heated to a high temperature under very low pressure; then when the charcoal had been cooled to the temperature of liquid air, 975 c.c. of gas could be admitted suddenly and the pressure measured as it varied during adsorption from 90 cm. to 0.0001 cm. To accomplish this a MacLeod gauge and barometer were built together as shown in the diagram (Fig. 1), and were joined to a glass tube *T*. To this tube

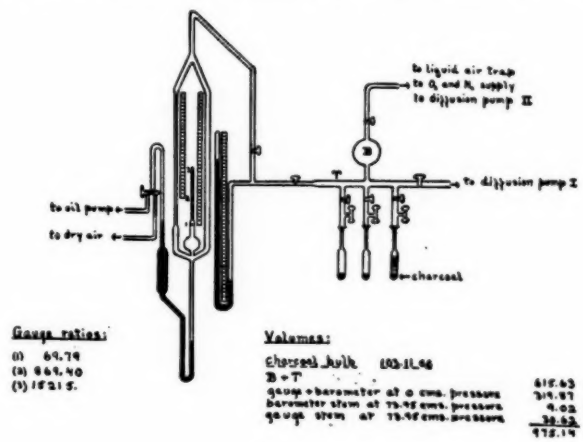

Fig. 1.

were also joined a half-liter glass bulb *B* to contain a large part of the gas before adsorption, and three Pyrex glass bulbs to hold the charcoal. The total volume of *B*, *T*, barometer and gauge was 975.14 c.c. Each charcoal bulb was sealed to a ground glass joint with sealing wax and was separated from *T* by a stopcock. A discharge tube joined on between the stopcock and bulb was useful to indicate any leak in the cock. During "outgassing" an electric oven was placed around the three bulbs, the temperature being indicated by a Leeds & Northrup potential

[1] Sir James Dewar, "The Adsorption and Thermal Evolution of Gases occluded in Charcoal at Low Temperatures," Proc. Roy. Soc., 74, 126, 1904.

point thermometer, and they were connected through T to a diffusion pump; during cooling the bulbs were shut off from T and each cooled in turn, just before using, by placing around it a container filled with liquid air. The bulb B was connected through a cock and through a liquid air trap to the oxygen and nitrogen supply and to a second diffusion pump which permitted thoroughly pumping one gas out of the connecting tube and trap before the other was admitted.

The charcoal used in the investigation was some that had been used previously in a long series of experiments described in a previous paper and was therefore completely activated and constant. Preliminary experimenting showed that 6.5 grams of charcoal was the amount needed

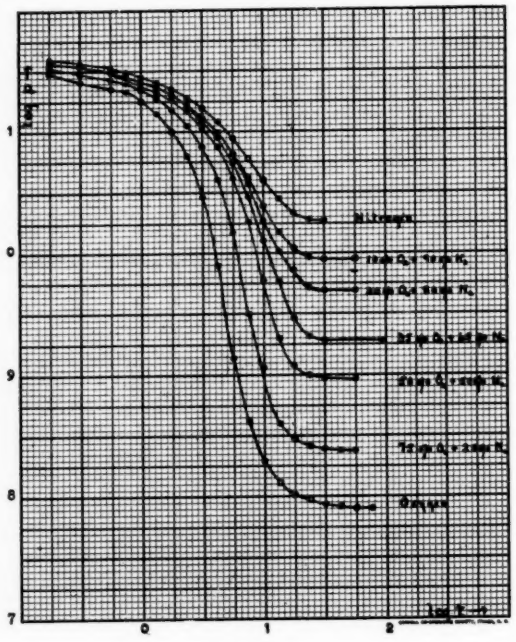

Fig. 2.

Adsorption of nitrogen, oxygen, and mixtures of the two gases.

to adsorb 975 c.c. of gas satisfactorily, for a larger amount gave such complete adsorption of both oxygen and nitrogen that the first requisite of the experiment, *i. e.*, a difference in the adsorption of the two gases, was lost.

Considerable precautions were taken to insure the purity of the gases used. Both oxygen and nitrogen were periodically examined spectroscopically and were used only when they were found to be of a high degree of purity. The oxygen was generated from pure peroxide of hydrogen,

sulphuric acid and potassium bichromate, then passed through caustic solution and a phosphorus pentoxide drying tube, and finally liquefied in the trap before being admitted to bulb *B*. The nitrogen was generated by heating a mixture of ammonium sulphate and sodium nitrate solutions. The gas was passed through both potassium bichromate and potassium hydroxide solutions, a P_2O drying tube, and the liquid air trap which completely removed all oxides of nitrogen.

In order to prepare the charcoal for adsorption by outgassing it, it was heated for four and a quarter hours at 582° C. That time has been found sufficient to expel by far the greater part of the gas in charcoal; and the temperature was as high as the walls of the charcoal tubes could be heated for a long period without collapsing. The pressure was

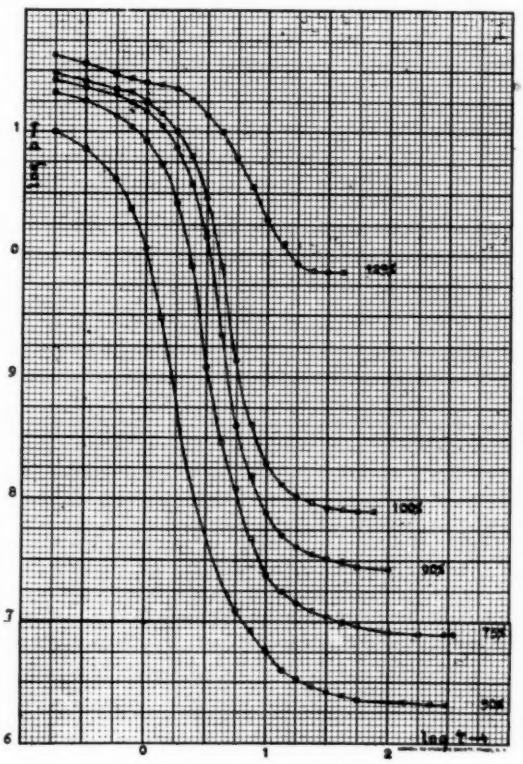

Fig. 3.

Adsorption of various quantities of oxygen alone.

read on the gauge at the completion of outgassing, the bulbs were shut off from the pump when they had cooled to 400, and the pressure was again read at 300 and at room temperature, to safeguard against possible leaks. At room temperature all traces of gas had disappeared, as far

as could be read from the gauge, and the three tubes were shut off from *T*.

The gas before being admitted to the charcoal was contained in *B*, *T*, the barometer and gauge, and its quantity measured by the pressure on the barometer. The same quantity of gas, 975.14 c.c. at 23° C. and 73.95 cm. (*i.e.*, 871.23 c.c. at 0° C. and 76.0 cm.) was used in all the experiments. Care was taken to insure complete mixture of the contents of *B* and *T* before testing.

Before admitting the gas to the charcoal, the charcoal bulb was tested with the discharge tube to make certain that no gas had leaked in from *T* since outgassing, and the bulb was cooled for at least twenty minutes in fresh liquid air. Precautions were taken to insure the freshness of the liquid air which was used in all cases when it was between two and four hours old, and to obtain full consistency in the results the same tube with the same charcoal was used in all the recorded experiments, the other two tubes being used to check results. When the tube was thoroughly cooled, the stopcock into *T* was opened quickly and as the gas rushed into the empty space and was adsorbed by the charcoal the rapidly decreasing pressure was read on the barometer and then on the gauge at intervals timed by a stop-watch. These intervals beginning with 11 seconds and ending with 1 hr. 40 mins. or later, according to the rate of adsorption, where chosen with a view to plotting logs. of pressure against logs. of times to obtain a compact and convenient curve, and were therefore so arranged that the logs. of times came at regular successive intervals. A typical record of an adsorption is shown in Fig. 3.

The series of curves obtained for pure oxygen, pure nitrogen, and five mixtures is shown in Fig. 2. It will be noticed that there is a slight hump at the beginning of the oxygen curve which appears again more strongly in the first oxygen curve in Fig. 3. This is due to the fact that the charcoal is at a temperature below the boiling point of oxygen which introduces conditions for the adsorption of considerable quantities of oxygen which do not exist in the other cases. The curves are regularly spaced at intervals which are not far from proportional to the percentage of one gas, say oxygen, in the mixture. To show the relation of the intervals to the percentages, in Fig. 5 (1) the logs of the final pressures in Fig. 2 have been plotted against percentages of oxygen. The result is a curve very slightly concave. It has been found, then, that the rates of adsorption of mixtures of oxygen and nitrogen vary continuously from the rate for nitrogen to that for oxygen, but that this variation takes place not according to a linear relation, but according to a relation that is very nearly exponential.

Numerical values are tabulated below:

TABLE I.

Mixtures of Oxygen and Nitrogen.

Oxygen, Per Cent.	Nitrogen, Per Cent.	Final Pressure.	Log p.
100		0.00797	7.902
75	25	.0240	8.380
50	50	.0924	8.966
35	65	.214	9.332
20	80	.489	9.690
10	90	.886	9.948
	100	1.86	0.270

To determine if the two gases acted independently when adsorbed as a mixture, the quantities of each gas present in certain mixtures were experimented upon separately. The system was filled with one gas only, at a pressure which was a given fraction of the usual 73.95 cm., and this gas was adsorbed as before. Fig. 3 shows the series of curves obtained

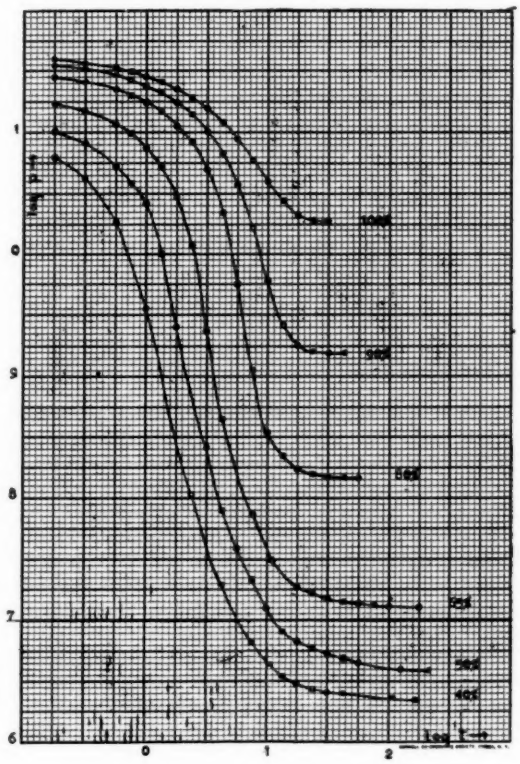

Fig. 4.

Adsorption of various quantities of nitrogen alone.

in this way for oxygen, including one taken with 125 per cent. of the usual quantity of gas. Fig. 4 shows a similar series for nitrogen. In Fig. 5, (2) and (3), the logs of these final pressures have been plotted

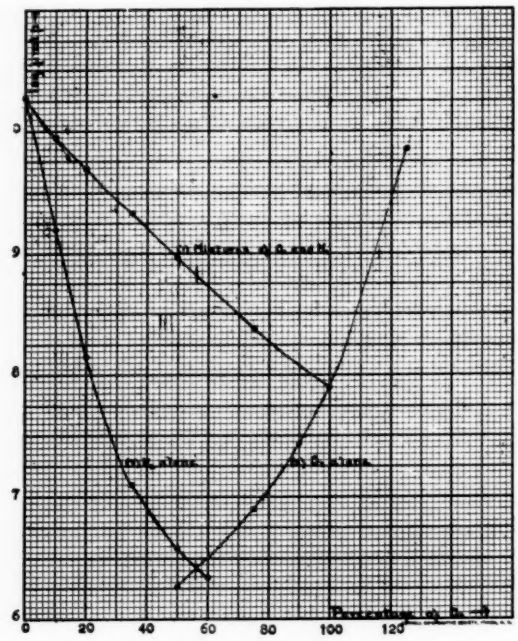

Fig. 5.

Variation of final pressure with gas adsorbed. Oxygen and nitrogen adsorbed separately 2) and (3); together in mixture (1).

against percentage of oxygen, decreasing nitrogen percentage being plotted as increasing oxygen percentage. Table II. contains the numerical results in these cases.

TABLE II.

Oxygen and Nitrogen adsorbed separately.

Oxygen, Per Cent.	Final Pressure.	Log p.	Nitrogen, Per Cent.	Final Pressure.	Log p.
125	0.715	9.854	100	1.86	0.270
100	0.00797	7.902	90	0.157	9.196
90	0.00269	7.430	80	0.0144	8.158
75	0.000785	6.895	65	0.00127	7.102
50	0.000181	6.259	50	0.000386	6.587
			40	0.000220	6.343

(*c*) RELATED PHENOMENA.

In the course of the work it became of interest to know if the final adsorption would be the same if a given amount of a pure gas were

adsorbed all at once as if half were adsorbed at one time and the second half admitted later. It was found, as the data will show, that adsorption is more complete in the second case.

	Final Pressure.	Log p.
(*a*) Oxygen, 100 per cent.	0.00797	7.902
(*b*) Oxygen, 50 per cent. 50 per cent.	0.00604	7.781
(*a*) Nitrogen, 100 per cent.	1.86	0.270
(*b*) Nitrogen, 50 per cent. 50 per cent.	0.649	9.812

In this connection it was suspected that charcoal exposed first to 50 per cent. oxygen and then to 50 per cent. nitrogen might yield different results from charcoal exposed in a manner vice versa. The difference was found to be too slight to have any significance, being within the range of experimental error; but as one would expect from the phenomena noticed above, the adsorption was more complete than in the case of a 50 per cent.–50 per cent. mixture of the two gases adsorbed all at one time.

	Final Pressure.	Log p.
1. (*a*) Oxygen 50 per cent., (*b*) nitrogen 50 per cent.	0.0451	8.654
2. (*a*) Nitrogen 50 per cent., (*b*) oxygen 50 per cent.	0.0481	8.682
3. Oxygen 50 per cent., nitrogen 50 per cent.	0.0924	8.966

(*d*) Conclusions Regarding Independence of Gases.

It is readily seen from a consideration of the curves in Fig. 5 that the two gases do not act independently in a mixture. For the logarithm of the sum of the final pressures of each component gives nowhere a point approaching curves (1). This is not an unnatural result, for if the interior surfaces of the charcoal are largely covered with one gas it is plausible to suppose that the forces producing adsorption are different from those that the denuded surface would show.

F. Bergter has stated as a result of his work on oxygen and nitrogen that[1] "In the range of pressure which I have investigated (low pressures of 1 mm. and less) the ability of charcoal to adsorb nitrogen must be increased by the presence of oxygen." He found that by using 9.483 gram of charcoal to adsorb aid at 0.523 mm. pressure, he obtained as a lowest estimate 59 units of nitrogen, while the same charcoal exposed to nitrogen alone adsorbed 29 units. The present work done under altogether different conditions of pressure, volume, and quantity of charcoal used, can neither verify nor refute this statement. But since in the present work the adsorption of a mixture was found inferior to the sum of the adsorptions of the components, it is not probable that Bergter's

[1] F. Bergter, "Der Zeitliche Verlauf der Adsorption von Gasen durch Holzkohle," Annalen der Physik, 37, 606, 1912.

discovery is applicable to this case, different from his own; for if the ability of the charcoal to adsorb the nitrogen were increased by the presence of the oxygen, then the presence of the nitrogen must to a far greater extent have hindered the adsorption of the oxygen. It seems simple and more natural to accept the hypothesis that each gas hindered the adsorption of the other.

RYERSON PHYSICAL LABORATORY,
March, 1919.

THE END CORRECTION IN THE DETERMINATION OF GAS VISCOSITY BY THE CAPILLARY TUBE METHOD.[1]

BY A. F. BENTON.

SYNOPSIS.

The voluminous data obtained for air from experiments on capillary tubes as flow meters have been used to test the formulae involving end-corrections, proposed by Brillouin and by Fisher. A measure of the validity of a given formula is obtained by assuming the value of the coefficient of viscosity and comparing the mass or gas transpired per second, as found experimentally, with the mass computed from the formula in question. The tubes employed in the tests ranged in size from very long lengths, where the end-effect was practically negligible, down to short lengths that they behaved as orifices.

Under this very exacting test, Fisher's equation for the end-correction is shown to be wholly unsatisfactory and to lead to impossible results. Brillouin's formula however reproduces the observed data fairly well. The agreement becomes excellent if the latter formula is slightly modified by the introduction of a constant, similar to the "orifice coefficient", to take account of the form of the ends of the tube. Finally a criterion is given for determining the negligibility of the end-correction within any desired limits of accuracy.

THE ordinary development of the equation for the flow of fluids through a capillary tube involves the use of the pressures inside the tube, at such a distance from the ends that the stream lines are parallel. The pressures measured in the relatively large end-vessels are greater than those demanded by the formula, since here the velocity of the fluid is practically zero, and the maximum pressure in a moving fluid is obtained at a point where the velocity is least. The possibility of error from this source in the capillary tube method of determining viscosity, was recognized very early. The older writers however have generally tried to make the error negligible by using very long, fine tubes, but in so doing have often introduced relatively great errors in the measurement of the radius.

The first attempt to set up an equation for the correction was made by M. Brillouin.[2] He considers that the kinetic energy of the emergent jet is a measure of the pressure lost at the ends of the tube, and, on this basis, derives the following modified form of Poiseuille's law:

$$p_1^2 - p_2^2 = \frac{p}{\rho}\left[\frac{16\eta LM}{\pi R^4\left(1 + \frac{4\xi}{R}\right)} + \frac{2M^2}{\pi^2 R^4}\right],$$

[1] Published by permission of the Director of the Chemical Warfare Service.

[2] Leçons sur la Viscosité des Liquids et des Gas, Vol. 1, p. 133; Vol. 2, p. 37.

where p_1 and p_2 are the entrance and exit pressures, respectively; p and ρ are the pressure and corresponding density of the fluid, measured under the same conditions; M is the mass transpired per unit of time; η is the coefficient of viscosity, and ξ the coefficient of slip; and L and R are the length and radius of the tube, respectively. More recently, W. J. Fisher[1] has put forward another formula for the correction, which he regards as a closer approximation to the true state of affairs than Brillouin's:

$$R'T ln \frac{p_1}{p_1'} = -\frac{1}{384}\left[\frac{p_1^2 - p_2^2}{\eta p_1 L}\right]^2 [R^4 + 12\xi^2 R^2 + 6\xi R^3].$$

Here R' is the gas constant, T is the absolute temperature, p_1' is the measured pressure in the vessel at the inlet end, and p_1 is the pressure to be used in Poiseuille's law. A similar equation with subscripts 2 gives the correction to the exit end. In applying this formula to his own data, Fisher finds the correction to be negligible.

Brillouin's correction has apparently been applied only by I. M. Rapp,[2] who has calculated the corrections to his data by both formulæ, and finds them in fairly good agreement. However, the writer is unable to check Rapp's calculations of the Fisher correction, except on the assumption that p_2 is greater than p_2', whereas the formula clearly shows that p_2 must be less than p_2'. It is hard to conceive how the pressure could be greater at a point where the velocity is high (p_2), than at a point where it is practically zero (p_2'). Fisher's correction, properly applied to the data, is negligible. Rapp's use of it has furnished values in fair agreement with Brillouin's formula in this case, only because the corrections are very small. An inspection of Fisher's equation will suffice to show that, on the assumption which Rapp appears to have made, p_2 could quite easily become larger than p_1, in which case the substitution of these values in Poiseuille's law would give a negative coefficient of viscosity. A further examination of the formula will make clear however that, even when properly applied, Fisher's correction can not be universally valid. For it will be observed that in any given trial,

$$ln \frac{p_2}{p_2'} = \frac{p_1^2}{p_2^2} ln \frac{p_1}{p_1'}.$$

If the pressure difference be made very large, it is evident that $ln \cdot p_2/p_2'$ will have a very much larger numerical value than $ln \cdot p_1/p_1'$, and p_2 becomes very small, so that its square is negligible in comparison with p_1^2.

[1] PHYS. REV., 32, 216, 1911.

[2] PHYS. REV., (2), 2, 363, 1913.

Poiseuille's law, corrected by Fisher's equation, then gives

$$M \propto \frac{p_1^2 - p_2^2}{p_2'} \propto \frac{p_1^2}{p_2'},$$

from which it follows that, since p_1 decreases as the measured pressure difference increases, M should now decrease as the pressure difference increases. A graph of Fisher's correction which illustrates this retroflex effect will be discussed below. This behavior is of course not found experimentally.

In Rapp's data the necessary corrections are so small that the results do not give a satisfactory test of any formula. He has nevertheless deduced an empirical correction from some of his data, concerning which it will only be pointed out here that it is open to the very serious objection that under certain conditions it can give negative values for the viscosity coefficient.

In connection with the use of capillary tubes as flow meters,[1] a considerable quantity of experimental material has been obtained by the author, which throws light on the end correction. The results, though not of the highest accuracy, are, as will appear from what follows, entirely adequate for the present purpose. Tubes of approximately 0.035 cm. and 0.07 cm. radius were employed, and tests were made on eight different lengths of each, from 1 m. down to 1 cm. Only those values have been used which lay below the critical region.[2] In testing the various formulæ against these results, which were all obtained with air, the coefficient of viscosity has been assumed known, and the weight of gas transpired per second is compared with the observed weight. Table I. gives a number of points selected at random. Space does not permit of including all the rather voluminous experimental material, but the magnitude of the correction in each case is indicated by the difference between columns 1 and 4. It is evident that Brillouin's formula is in fair agreement with the observed values, but Fisher's correction does not even approximately represent the data. These facts are strikingly shown in the accompanying figure, in which the radius was approximately 0.034 cm. and the length was only 0.93 cm. The circles indicate observed points. The retroflection of the curve for Fisher's correction, anticipated above, is here very well illustrated. This extreme case has been chosen solely for convenience in graphical representation.

[1] These are described in a paper on "Gas Flow Meters for Small Rates of Flow," J. Ind. Eng. Chem., 11, 623, 1919.

[2] The location of the critical region was ascertained by plotting all the results in the usual form, suggested by an application of the "principle of dimensional homogeneity." See for example, T. E. Stanton and J. R. Pannell, Trans. Roy. Soc. London, A, 214, 199, 1914.

The same result is obtained with longer tubes, but in that case the vertex of the curve lies at a higher value of the pressure difference.

Although Brillouin's formula represents all the results fairly well, it gives values for the mass transpired which are higher than the observed values by 3 to 5 per cent. This deviation greatly exceeds the experimental error, and is always in the same direction. The discrepancy is undoubtedly to be accounted for by the fact that all the tubes here used had sharp ends, which produce a very considerable contraction in the cross section of the gas stream. This means in effect that the radius in the last term of Brillouin's formula is smaller than the radius in the preceding term. If C be used to denote the ratio of the radii of the tube and of the vena contracta, respectively, Brillouin's equation becomes

$$p_1^2 - p_2^2 = \frac{p}{\rho}\left[\frac{16\eta LM}{\pi R^4\left(1 + \frac{4\xi}{R}\right)} + \frac{2CM^2}{\pi^2 R^4}\right].$$

TABLE I.

Mass of Air Transpired, in Milligrams per Second.

Mass of Air Transpired, in Milligrams per Second.			
Observed.	Calculated by the Formula of		
	Brillouin.	Fisher.	Poiseuille.
2.23	2.35	2.35	2.38
4.66	4.96	6.27	6.32
8.36	8.90	27.61	27.80
17.29	17.72	75.96	92.41
21.00	21.44	73.84	130.2
25.77	26.64	32.03	194.6
26.38	27.20	28.75	29.20
38.44	39.25	125.9	126.5

C will in general depend on the radius and on the pressure drop, but within fairly narrow limits, it may reasonably be assumed constant, as is evidenced by the fact that the equation in this form represents the observed results very accurately, even when the end-correction is large. C will of course also be a function of the form of the ends of the tube, and if the latter are well rounded out, will be very close to unity, so that in this case it may probably be disregarded altogether, especially since the whole end-correction is ordinarily quite small.

An equation which is essentially a simplified form of Brillouin's has been developed during the course of the experiments, on the assumption that the total pressure difference across the capillary tube consists of

two parts, the frictional resistance within the tube and the resistance of the orifices at the ends. If the latter be denoted by h_0, the former by h_L, and the total resistance by H, a combination of Poiseuille's law with an approximate expression[1] of the orifice formula gives

$$H = h_L + h_0 = \frac{k_1 \eta LM}{\rho R^4} + \frac{k_2 CM^2}{\rho R^4},$$

or

$$\rho H R^4 = k_1 \eta LM + k_2 CM^2.$$

k_1 and k_2 are constants, and C is the familiar orifice "coefficient." Since this equation is not strictly valid, and since, with sharp ends, C is a

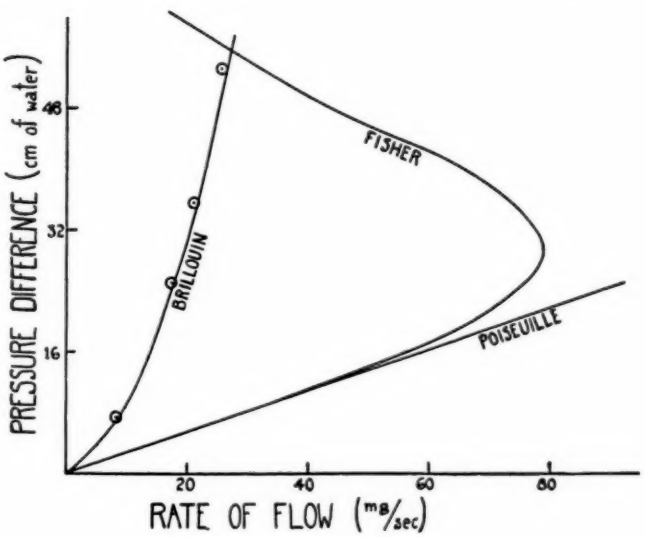

Fig. 1.

function of H and R, the values of k_1 and k_2 were determined experimentally rather than theoretically. Application of the method of least squares to the forty-nine observations which were below the critical region, gave

$$\rho H R^4 = 2.580 \eta LM + 0.1248 M^2.$$

This equation represents the observed results, even in the most extreme cases, where the experimental error is relatively great, within 2 per cent. Where high pressure differences are used, it will of course give only a rough approximation. In such cases the more exact form given by Brillouin should be employed.

[1] This simplification is permissible here because relatively small pressure differences were employed. See R. J. Durley, Trans. A. S. M. E., Vol. 27, p. 193, 1906.

From these equations it is a very simple matter to derive a criterion for the negligibility of the end-correction, within any desired accuracy. For example, if the correction is to be less than 0.1 per cent., it is evident that $2.580\eta ML$ must be greater than $1{,}000 \times 0.1248M^2$. That is, $\eta L/M$ must be greater than 48.4. Since $\eta L/M$ is a dimensionless product, its value is independent of the system of units in which it is expressed, as long as their interrelations remain unchanged. In fact Brillouin's equation and the simplified form of it are each equally valid whether metric or English units are employed.

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THE RADIATING POTENTIALS OF NITROGEN.

BY H. D. SMYTH.

SYNOPSIS.

1. A formula is derived by which accurate corrections allowing for the distribution of velocities of the impacting electrons may be applied to the observed values of the radiating potentials.

2. Measurements on nitrogen revealed—

(*a*) a very strong effect at 8.29 ± 0.04 volts.

(*b*) a very doubtful effect at 7.3 volts.

(*c*) an effect appearing only at lower pressures but strongly at 6.29 ± .06 volts.

3. These results are explained as follows:

(*a*) The wave-length λ corresponding to 8.29 ± .04 is 1490.7 ± 10 and this effect is ,therefore, identified with the doublet found by Lyman at 1492.8 and 1494.8.

(*b*) This gives λ = 1700 and may be identified with the second doublet 1742.7 and 1745.3 attributed to nitrogen but by some thought due to silicon.

(*c*) The value λ = 1965 ± 20 from 6.29 ± .06 volts is taken to correspond to the beginning of the band spectrum at 1870.9. The discrepancy is attributed to the presence of nitrous oxide.

According to another theory the effect at 6.29 is considered due to lines in the region 2,000–3,000 A.U. coming from neutral atoms. The value 6.29 in this case is taken as the speed necessary before the electrons can split up the molecules.

4. From 3 (*a*) assuming line spectra to come from atoms, we have as an upper limit to the heat of dissociation of a gram molecule of nitrogen 190,000 calories.

From the second theory in 3 (*b*) we have as a possoible actual value for this heat of dissociation 145,000 calories.

5. Qualitative evidence was obtained supporting Davis and Goucher's discovery of true ionization in the neighborhood of 18 volts.

I. INTRODUCTION.

THE substances whose minimum radiating and ionizing potentials have been investigated fall naturally into two classes, first, the monatomic gases and metallic vapors, in which collisions below a certain velocity are elastic, and second, the diatomic gases, in which collisions at all velocities are inelastic or at least partially so, as in the case of hydrogen.

The investigations for substances of the first class have recently been summarized and discussed by McClennan.[1] The conclusions drawn are, briefly, as follows. A vapor, when bombarded by electrons of velocity V emits a radiation $eV = h\nu$. The first radiation produced is the line

[1] J. C. McClennan, "The Origin of Spectra," Proc. London Phys. Soc., XXXI., Part I., pp. 1–29, 1918.

at the head of the single t principal series, and thereafter shorter and shorter wave-length radiations set in as the bombarding electrons attain correspondingly higher velocities. Finally, when the speed of the electrons reaches a value corresponding to the frequency $\nu = (1.5, S)$, the convergence frequency of the singlet principal series of the element in question, ionization occurs.

For diatomic gases the mechanism of ionization or the production of radiation is obviously more complicated than where only a single atom is involved. Either it is necessary first to dissociate the molecules and then, by a second impact, to cause radiation or ionization, or only one collision is necessary. Again, if the effect is the result of one impact, it may occur in two ways; an electron may be displaced and the molecule broken up simultaneously, or an electron may be displaced without affecting the bond between the atoms.

Possibly as a result of the above, the spectra of the diatomic gases are very complex and, for the most part, not resolved into series. This makes the prediction of ionizing and radiating potentials difficult. Another source of error especially serious in the case of diatomic gases arises from the uncertainty in applying the necessary correction for velocity distribution, as discussed later in the paper.

In consequence of these complications, although values of the ionizing and radiating potentials have been determined for a number of different diatomic elements and compounds, little has been accomplished toward connecting them with the spectra. In spite of the simple structure of the hydrogen molecule and the extensive data on the spectrum available, the experimental results obtained have not yet been reconciled with those predicted by Bohr's theory or by the quantum relation[1] $eV = h\nu$.

Though the spectrum of nitrogen has not been resolved into series, in the region of the ultra-violet with which we are concerned the line spectrum is very simple. Lyman,[2] in his investigation of the extreme ultra-violet, found for nitrogen no lines except two doublets, one consisting of the lines 1492.8 and 1494.8 and the other[3] of the lines 1742.7 and 1745.3. Applying the quantum relation to these, we have for the corresponding radiating potentials, 8.28 and 8.27, and 7.04 and 7.08 volts. The longest wave-length of the band spectrum which Lyman found in this region is 1870.9 corresponding to 6.6 volts. Now the generally accepted value for the radiating potential of nitrogen is 7.5

[1] Bergen Davis and F. S. Goucher, PHYS. REV., No. 10, p. 101, 1917.

[2] Lyman, "Spectroscopy of Extreme Ultraviolet," p. 113, 1914.

[3] Possibly due to silicon.

volts while Davis and Goucher[1] found a second more intense radiation at 9 volts.[2]

In view of the discrepancy between these experimental values and those calculated from the spectrum, it was thought worth while to attempt a more exact determination.

II. APPARATUS.

The apparatus used had been designed for a somewhat different purpose but was found fairly satisfactory.

As shown in Fig. 1, it consisted of a glass tube 1.5″ in diameter, with a filament *F* sealed in at one end, a gauze *G* in the middle and a disc *P* sealed in from the other end, all of platinum. Electrical connections

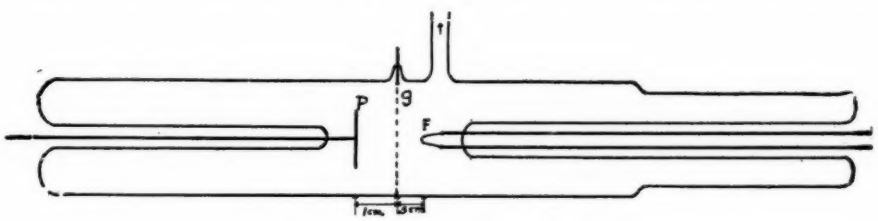


Fig. 1.

were so arranged that the electrons coming off from the hot filament *F* were accelerated by a field between *F* and *G* but met a stronger retarding field between *G* and *P*. The difference between accelerating and retarding fields remained constant. The gauze *G* was connected to a Leeds & Northrup high sensitivity galvanometer which measured the electronic current between *F* and *G*. The disc *P* was connected to a Dolazalek electrometer of sensitivity about 3,000 millimeters per volt, making the capacity of the electrometer system of the order of magnitude of 50 cm. The potentials were read on a Robt. W. Paul voltmeter.

By coating the filament with barium oxide increased emission was obtained.

The pressure in the apparatus was regulated by a Gaede pump and measured with a McLeod gauge.

Nitrogen was prepared by heating sodium nitrite and ammonium chloride with distilled water and was introduced through a drying tube. The apparatus was evacuated and washed out several times with nitrogen before measurements were made.

The setting in of radiation was detected in the usual manner by the photoelectric effect on *P* causing an increase in the speed of deflection of

[1] Bergen Davis and F. S. Goucher, PHYS. REV., No. 13, pp. 1–5, 1919.

[2] The fact that these are radiation effects and not ionization seems to have been proved by Davis and Goucher and is taken for granted in this paper.

the electrometer. It was found impossible to eliminate all zero drift from the electrometer. This made it necessary to start readings far enough below the break point to give a good zero. The accelerating potential was run from 3 or 4 volts up to 11 or 12 and then down again, the intervals between readings near critical points being as small as .2 of a volt. The values going up and coming back were averaged. The electronic current measured by the galvanometer was kept constant in the earlier runs by adjusting the temperature of the filament but, in the later runs, the temperature of the filament was maintained constant and the galvanometer current allowed to vary with the accelerating potential, thus tending to accentuate the sharpness of the break.

III. Velocity Distribution Correction.

The most serious source of error in experiments of this type is due to the fact that the electrons reaching the gauze will not all have exactly the velocity corresponding to the accelerating field.[1] The factors causing this trouble are the potential drop along the filament, initial velocity of emission and, in the case of diatomic gases, inelastic impacts.

Now, in the case of monatomic gases the elastic impacts make possible a very effective method of eliminating this error. In this case, the electrometer current rises rapidly as the accelerating field passes the critical value, reaching a maximum when all the electrons have attained the critical speed, and then falls off again as more and more of the ionizing collisions take place on the filament side of the gauze. When the accelerating potential becomes great enough to allow two ionizing collisions by one electron another maximum occurs, and so on. Thus, by measuring the intervals between successive maxima the true value of the ionizing potential can be found.[2]

With diatomic gases, however, this method is impossible since, with every increase of the accelerating field, some electrons which have lost energy by inelastic impact will attain the critical speed and there will be a continuous increase in the electrometer current. It becomes necessary, therefore, to actually measure the velocity of the electrons coming across and then make a correction.

Franck and Hertz did this but felt so uncertain as to the right method of making the correction that they claimed an accuracy of only one volt[3] for their results.

Goucher[4] eliminated errors due to the potential drop along the filament

[1] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 15, p. 37, 1913.
[2] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 16, p. 457, 1914.
[3] J. Franck and G. Hertz, Verh. d. D. Phys. Ges., 15, p. 39, 1913.
[4] F. S. Goucher, PHYS. REV., No. 8, p. 561, 1916.

by introducing an equipotential electron source consisting of a platinum thimble surrounding a tungsten heating element. With this arrangement in mercury vapor he found that over 70 per cent. of the electrons had velocities corresponding to the applied voltage and, further, that the number having a velocity corresponding to .5 volt greater than the applied field was too small to be measured. He therefore made no correction for velocity distribution. In the later work of Davis and Goucher[1] on nitrogen, the same type of electron source was used and the correction considered unnecessary. As before stated the values obtained were 7.5 and 9 volts.

Bishop,[2] working with a filament, took the point of maximum slope of his electron current curve for his velocity distribution correction. That is, he took the most probable velocity of the electrons. He found the value of 7.5 volts for nitrogen and the same for N_2O.

Hughes and Dixon,[3] who obtained the values 7.7 volts for nitrogen and 9.3 for nitric oxide, took as their velocity correction the highest speed detectable on their velocity distribution curve.

Attempts to apply velocity distribution corrections in preliminary tests of the present apparatus proved that the methods mentioned above give quite different results, and that the discrepancy between them varies with the filament temperature and gas pressure. Furthermore, when correcting by the method of Hughes and Dixon, the relative sensitivities of the apparatus for radiation and for velocity distribution measurements and, also, the scale to which the measurements were plotted could be altered so as to vary the value of the corrected "break" point by as much as a volt, without any obvious way of selecting the correct value from among the various possible ones.

This experience led to a closer study of the problem with a view of determining the method of handling data from the two types of measurement which would give the most nearly correct result. The variation of zero drift sets a practical limit to the scale of plotting of either curve. Therefore, it remains to determine an appropriate scale for the more sensitive measurement and this is done by finding the relation between the sensitivities of the apparatus for the two types of measurement. Obviously, the "break" point in the radiation curve is due to the radiation from the smallest number of electrons which can produce a large enough radiation effect to be detected by the apparatus. Since not all electrons capable of producing radiation do so, owing to failure to collide or for other reasons, this "smallest" number of electrons is larger than

[1] Bergen Davis and F. S. Goucher, PHYS. REV., No. 13, p. 1, 1919.

[2] F. M. Bishop, PHYS. REV., No. 10, p. 244, 1917.

[3] A. Ll. Hughes and A. A. Dixon, PHYS. REV., No. 10, p. 495, 1917.

the least number which can be detected if they are permitted to strike the receiving electrode, as in the velocity distribution measurements. The problem, therefore, is to find how many electrons must pass the gauze with sufficient energy to cause radiation in order that one may produce radiation. In other words, how many times less sensitive is the apparatus for radiation than for velocity distribution measurements and how should the latter be plotted in order that the greatest velocity shown should give the correction appropriate to the "break" point in the former?

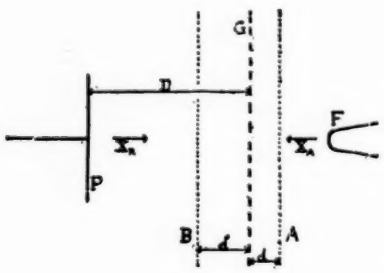


Fig. 2.

The following analysis leads to a very usable expression for the ratio of the sensitivity of the electrometer system for radiation to that for velocity distribution experiments.

Let:

N = number of collisions per centimeter path at 1 mm. pressure;

X_A and X_R = the electric intensities in the accelerating and retarding fields, respectively;

n = number of electrons per unit time reaching the gauze G from the filament side;

$f(V)dV$ = the probability of an electron reaching the gauze with a speed between V and $V + dV$;

V_0 = the minimum radiating velocity;

A and B be two planes parallel to G distant d and d' on either side of it, where

$$d = \frac{V - V_0}{X_A}$$

and

$$d' = \frac{V - V_0}{X_R};$$

$f(V_0)dV$ = the probability of an electron reaching A with velocity between V_0 and $V_0 + dV$.

In all cases, velocities are expressed in terms of equivalent volts.

Evidently

$$nf(V_0)dV(1 - \epsilon^{-pNd}) = n(\epsilon^{pNd} - 1)f(V)dV$$

is the number of electrons which pass A with velocities between V_0 and $V_0 + dV$ and collide before reaching G.

After passing the gauze the electrons will go a distance

$$d' = (V - V_0)/X_R$$

before losing their ability to cause radiation. From these we have

$$nf(V)dV(1 - \epsilon^{-pNd'})$$

as the number of radiating impacts after passing the gauze by electrons which reached the gauze with speeds between V and $V + dV$.

We have, then, for the total number of impacts at a speed greater than V_0, by electrons going in the direction away from the filament,

$$M_1 = n\int_{V_0}^{V_m} (\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_R}})f(V)dV, \tag{1}$$

where V_m is the maximum speed of any electron reaching G.

There will also be a small number of electrons which will go through, be stopped, and acquire sufficient acceleration in the opposite direction to produce radiation. For these we have

$$n' = n\epsilon^{-pN\left(\frac{V-V_0}{X_R}\right)}f(V)dV$$

as the number which do not collide before their velocity is retarded to V_0, and

$$n'' = n'\epsilon^{-pN\frac{2V_0}{X_R}}$$

as the number which will regain a velocity V_0 in the opposite direction before collision.

Then

$$n''(1 - \epsilon^{-pN(d'+d)})$$

is the number which, having passed G with velocity between V and $V + dV$, escape collision until they have reversed their direction and regained a speed greater than V_0, finally colliding before their speed is again reduced to V_0.

On substitution we have

$$\begin{aligned} dM_2 &= n'\epsilon^{-pN\frac{2V_0}{X_R}}(1 - \epsilon^{-pN(d'+d)}) \\ &= n\epsilon^{-pN\frac{2V_0}{X_R}}\cdot\epsilon^{-pN\left(\frac{V-V_0}{X_R}\right)}[1 - \epsilon^{-pN\left(\frac{V-V_0}{X_R}+\frac{V-V_0}{X_A}\right)}]f(V)dV \qquad (2) \\ &= n[\epsilon^{-pN\frac{V+V_0}{X_R}} - \epsilon^{-pN\left(\frac{2V}{X_R}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \end{aligned}$$

Therefore

$$M_2 = n\int_{V_0}^{V_m} [\epsilon^{-pN\frac{V+V_0}{X_R}} - \epsilon^{-pN\left(\frac{2V}{X_R}+\frac{V-V_0}{X_A}\right)}]f(V)dV \tag{3}$$

is the number of collisions by electrons coming in the reverse direction and colliding while they have a velocity between V_m and V_0.

We have, for the total number of collisions by electrons having velocities greater than V_0,

$$M = n\int_{V_0}^{V_m}[\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_R}}]f(V)dV + n\int_{V_0}^{V_m}[\epsilon^{-pN\frac{V+V_0}{X_R}} - \epsilon^{-pN\left(\frac{2V}{X_R}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \quad (4)$$

Now if we take the probability of the production of radiation and its detection by photoelectric effect to be $(V - V_0)/kV_0$, where the value of kV_0 may be found from the results of Johnson[1] we have for the effective number of radiating impacts when the maximum electronic speed is V_m,

$$M' = n\int_{V_0}^{V_m}\frac{V-V_0}{kV_0}[\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_R}}]f(V)dV + n\int_{V_0}^{V_m}\frac{V-V_0}{kV_0}[\epsilon^{-pN\frac{V+V_0}{X_R}} - \epsilon^{-pN\left(\frac{2V}{X_R}+\frac{V-V_0}{X_A}\right)}]f(V)dV. \quad (5)$$

We now turn to the velocity distribution measurements. If D is the distance from the gauze G to the plate P and V_R is the retarding potential, the number of electrons getting to the plate will be, for given values of V_m (*i.e.*, V_A, the accelerating potential) and V_R,

$$M_P = n\int_{V_R}^{V_m}\epsilon^{-pND}f(V)dV. \quad (6)$$

Now, the form of the function $f(V)$ will vary with V_m (or V_A) and its value will depend also on the particular value of V substituted.

For a given value of V_m we will have, from (6),

$$\frac{dM_P}{dV_R} = -n\epsilon^{-pND}f(V)_{V=V_R}. \quad (7)$$

Consider the part near the foot of a typical velocity distribution curve, as shown by Fig. 3. The curve is a section of a parabola, while actual experimental measurements are indicated by dots. It is evidently sufficiently accurate, for the present purposes, to consider the lowest part of the distribution curve to be parabolic, so that its slope is taken to be proportional to the horizontal distance from the foot. Use of this property gives the graph shown in Fig. 4.

Let $V_{R'}$ be some particular value of V_R; then

$$(M_P)_{V_R=V_{R'}} = -\tfrac{1}{2}(V_m - V_{R'})\left(\frac{dM_P}{dV_R}\right)_{V_R=V_{R'}}.$$

Also, we have, for any value of V_R less than V_m,

$$\frac{dM_P}{dV_R} = \frac{V_m - V_R}{V_m - V_{R'}}\left(\frac{dM_P}{dV_R}\right)_{V_R=V_{R'}}.$$

[1] J. B. Johnson, PHYS. REV., X., p. 609, 1917.

Combining these two equations, we have

$$(M_P)_{V_{R'}} = -\frac{1}{2}\frac{(V_m - V_{R'})^2}{V_m - V_R}\frac{dM_P}{dV_R}.$$

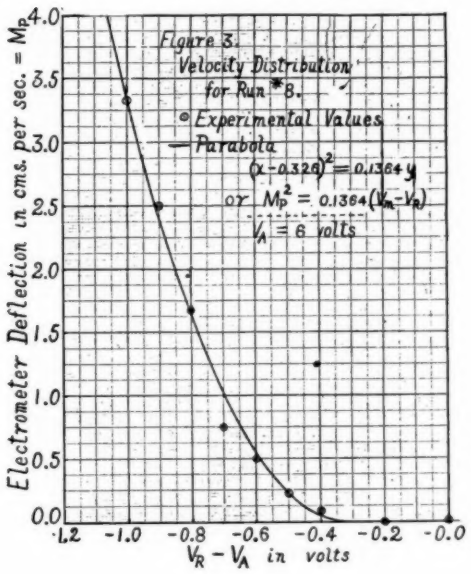


Fig. 3.

Substituting for dM_P/dV_R from (7), and solving for $f(V)_{V=V_R}$, we have

$$f(V)_{V=V_R} = \frac{2(M_P)_{V_{R'}}(V_m - V_R)\epsilon^{pND}}{n(V_m - V_{R'})^2}.$$

We are endeavoring to find the ratio of corresponding values of M'

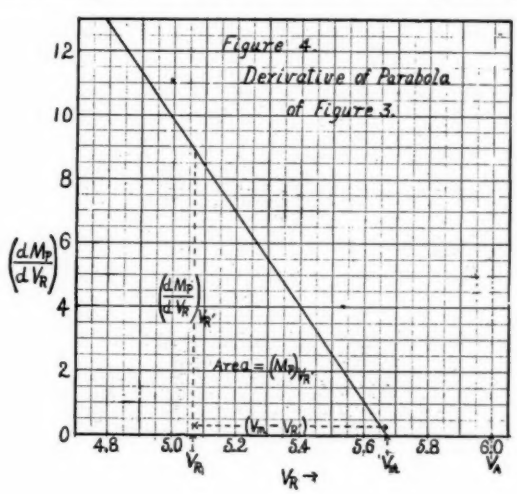


Fig. 4.

and M_P, *i.e.*, of values at equal distances from the break point. For this condition we must have

$$V_m - V_0 = V_m - V_{R'}$$

and $(V - V_0)$ in the case of radiation corresponds to $(V_m - V_R)$ in the velocity distribution. We can therefore substitute the value found above for $f(V)$ in the expression for M' if we write $V_m - V_0$ for $V_m - V_R$ and $V - V_0$ for $V_m - V_R$. If at the same time we divide through by $(M_P)_{V_{R'}}$, we have the following expression for the required ratio:

$$\frac{M'}{M_P} = 2\epsilon^{pND}\int_{V_0}^{V_m} \frac{1}{(V_m - V_0)^2}\frac{(V - V_0)^2}{kV_0}\left[\epsilon^{pN\frac{V-V_0}{X_A}} - \epsilon^{-pN\frac{V-V_0}{X_R}} + \epsilon^{-pN\frac{V+V_0}{X_R}} - \epsilon^{-pN\left(\frac{2V}{X_R}+\frac{V-V_0}{X_A}\right)}\right]dV$$

for corresponding values of M' and M_P in the neighborhood of the break point.

If, in the above expression, we put the constant factors outside the integral and set $pN/X_A = a$, $pN/X_R = a'$ and $V - V_0 = x$, and if we take $a = a'$, which is very nearly true we have

$$\frac{M'}{M_P} = \frac{2\epsilon^{pND}}{kV_0(V_m - V_0)^2}\int_0^{x_m} x^2[\epsilon^{ax} - \epsilon^{-ax} + \epsilon^{-2aV_0}(\epsilon^{-ax} - \epsilon^{-3ax})]dx.$$

By integration and the expansion of the powers of ϵ (either before or after integration), this equation takes the following form, which includes all terms as far as those in a^3:

$$\frac{M'}{M_P} = \frac{2a\epsilon^{pND}(V_m - V_0)^2}{kV_0}[1 - aV_0 - \tfrac{4}{5}a(V_m - V_0) + \tfrac{7}{9}a^2(V_m - V_0)^2 + \tfrac{16}{5}a^2V_0(V_m - V_0) + 2a^2V_0^2 + \cdots].$$

Practically, the terms in a^2 are negligible, since a is of the order of magnitude of 0.03.

If, therefore, we call the sensitivity of the apparatus for radiation R and for velocity distribution S, we have, for practical purposes,

$$\frac{R}{S} = \frac{M'}{10M_m} = \frac{\epsilon^{pND}a(V_m - V_0)^2}{5kV_0}(1 - aV_0) = \frac{\epsilon^{pND}a(V_m - V_R)^2}{5kV_0}(1 - aV_0), \tag{8}$$

where V_m is the maximum speed detectable in velocity distribution measurements and the factor 1/10 is introduced because, in our apparatus, the dimensions were such that P receives only about that proportion of the radiation starting at the gauze.

Equation (8) is evidently not exact, owing to the approximations

introduced in order to obtain a solution. It is believed, however, that none of these approximations is seriously in error, so that the results given by (8) should be of the right order of magnitude.

The necessary procedure, therefore, is to plot the velocity distribution on the same scale on which the radiation curve is to be plotted and determine V_m from this. Then multiply the values of the electrometer current for each value of V_R by the corresponding values of R/S and replot. The break point in this new curve determines the correction to be used on the radiation curve. The necessity of plotting a corrected velocity distribution curve is made evident by Table I. which gives the values of S/R calculated by equation (8) for different values of $V_R - V_m$, taking $k = 2$ for nitrogen.[1]

TABLE I.

$p = 0.015$ mm. $V_0 = 7.5$ volts.

$(V_m - V_R)$.	S/R.	$(V_m - V_R)$.	S/R.
.1	795,000	1.1	6,500
.3	87,500	1.2	5,500
.5	31,000	1.3	4,700
.6	22,000	1.4	4,050
.7	16,000	1.5	3,500
.8	12,500	1.6	3,200
.9	9,500	1.7	2,750
1.0	8,000		

IV. EXPERIMENTAL RESULTS.

It was found impossible to eliminate all zero leak from the electrometer. Consequently readings had to be taken at low accelerating potentials to establish a zero line. Errors due to a variation in this zero leak as well as errors of observation were pretty well eliminated by the method of taking a series of readings with fields increasing and then decreasing and plotting the mean. A typical set of readings is given in Table II., below.

where

V_f = P.D. across filament in volts;

G = current to gauze in amperes $\times 10^{-8}$;

T = time in seconds for electrometer to deflect one cm.;

$\overline{T}$ = mean of T and T' observations as V_A was increased and then decreased respectively;

$1/\overline{T}$ = rate of deflection of electrometer in cms. per second.

The values of V_A and $1/\overline{T}$, that is the accelerating potential and the current to the electrometer, were used in making the curves. The first

[1] J. B. Johnson, *loc. cit.*

TABLE II.

Observations for Run No. 12.

p = 0.01 mm., $V_R - V_A$ = 2.5 volts, V_f = 0.8 volts.

V_A.	G.	G'.	T.	T'.	$\overline{T}$.	$I/\overline{T}$.
1	6	6	81	81	81	.0124
2	16.4	17	79	86	82.5	.0121
3	21.2	21	76	86	81	.0124
4	23	22	82	80	81	.0124
5	23.7	22.4	73	89	81	.0124
5.5	24.1	22.5	71	74	72.5	.0138
5.75	24.4	22.4	76	71	73.5	.0136
6	24.7	22.3	66	61	63.5	.0158
6.25	25	22.1	63	60	61.5	.0163
6.5	25.2	22.1	60	56	58	.0172
6.75	25.5	21.9	55	55.6	55.3	.0181
7	25.8	21.7	50	53	51.5	.0194
7.25	26.2	21.6	46.4	45.4	45.9	.0218
7.5	26.6	21.5	44	44.6	44.3	.0220
7.75	26.9	21.3	41	41	41	.0244
8	27.2	21	35.2	35	35.1	.0285
8.25	27.4	21	29.8	30.2	30	.0330
8.5	27.7	20.9	23.4	27.5	25.5	.0392
8.75	28	20.8	20.2	22.1	21.1	.0474
9	28.2	20.7	16.8	19.4	18.2	.0550
9.5	28.5	20.8	10.6	12.5	11.5	.0870
10	28.7	20.8	7.5	9.0	8.2	.122
11	29.2	21	1.8	3.0	2.4	.417
12	29.5	21.1	.58	.7	.64	1.563

detectable departure from the zero line was taken as the break point. Thus, in run no. 4, Fig. 5, the break point comes at 9.3 volts. In order to correct this we examined the velocity distribution curve for run no. 4 in Fig. 6. Here we saw that the uncorrected values (curve *a*) gave a break point at $V_R - V_A = 0$ and therefore $V_m - V_A = 0$. Using this value to get $(V_m - V_R)$, and so to calculate the corrected values of the current by applying equation 8, we got the curve *b* which has its first break point at $V_R - V_A = -1.0$. We concluded, therefore, that the first electrons which are effective in producing detectable radiation have a velocity one volt less than the applied field, giving us 8.3 as the true value of the radiating potential from this run.

Similarly for run no. 5 we found the observed break to be at 9.0 and the corrected value to be 8.0 volts.

In run no. 6, Fig. 7, on the other hand, we found the corrected value to come at 6.5 (5.9 observed), but there is a sharp increase in the slope

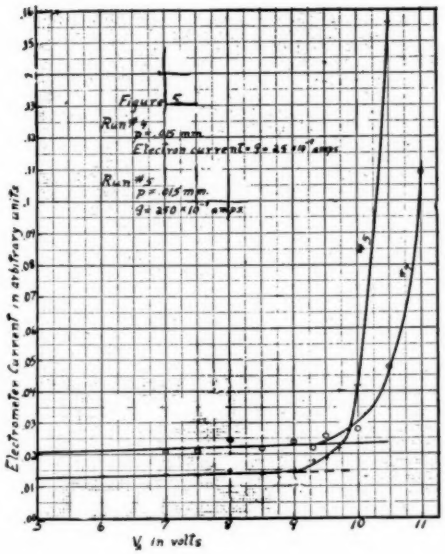


Fig. 5.

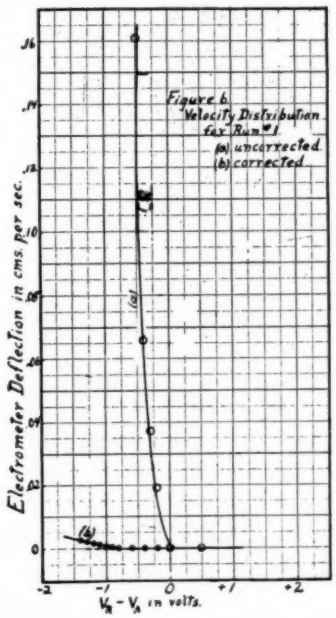


Fig. 6.

of the curve at 8.6 (8.0 observed). This run therefore apparently has two breaks corresponding to two critical speeds.

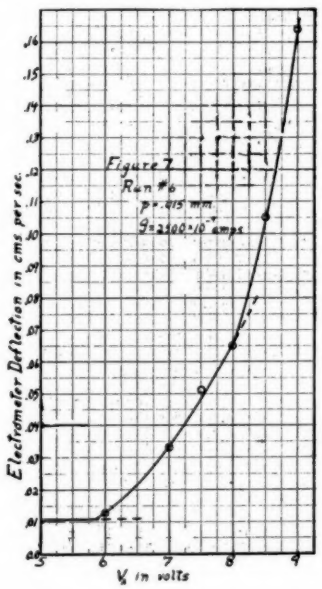


Fig. 7.

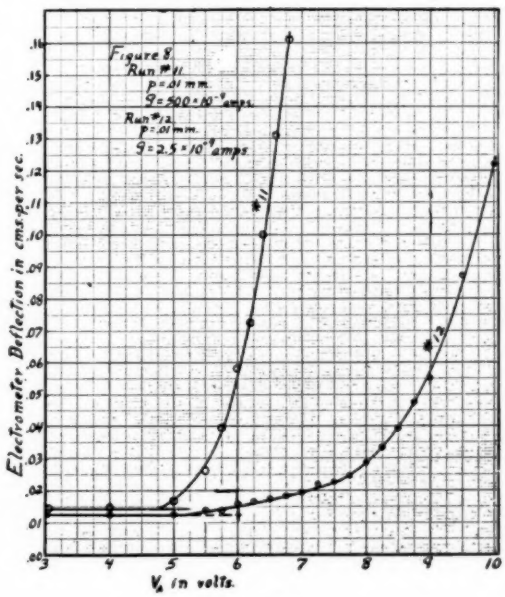


Fig. 8.

In runs nos. 11 and 12 the effect at the lower voltage was so great as to make it impossible to detect any other. For these runs the corrected values are 6.15 and 6.4 volts.

From these five typical curves shown, it is clear there are two distinct critical points, each curve showing one or both more or less sharply. The values observed were weighted according to the sureness with which the break point could be picked. The velocity distribution curves were treated similarly and the weight of the corrected value taken as the product of the weights of the two observations.

In Table III. the results from the curves shown and discussed above are grouped with all other runs which gave results sufficiently definite to have weight.

TABLE III.

Experimental Results.

No.	p (mm.).	G.	Observed Breaks with Weighting.		Corrections with Weighting.	Corrected Breaks with Weighting.	
1	.048	25.	9.4 (4)		−0.4 (3)	8.5 (12)	
2	.038	25.	9.2 (3)		−0.9 (3)	8.3 (9)	
3	.027	25.	9.3 (2)	7.5 (?)	−1.0 (3)	8.3 (6)	
4	,015	25.	9.3 (3)		−1.0 (3)	8.3 (9)	
5	.015	250.	9.0 (4)		−1.0 (3)	8.0 (12)	
6	.015	2500.	8.0 (1)	5.9 (2)	+0.6 (3)	8.6 (3)	6.5 (6)
7	.015	2500.		5.4 (5)	+0.7 (2.5)		6.1 (8)
8	.045	750.	9.3 (2)		−.08 (4)	8.5 (8)	
9	.045	140.	9.2 (1)		−1.0 (4)	8.2 (4)	
10	.026	250.	8.8 (2)	7.4 (1)	−0.7 (5)	8.1 (10)	6.7 (5)
11	.01	500		4.75 (4)	+1.4 (3)		6.15 (12)
12	.01	2.5	7.75 (?)	5.2 (4)	+1.2 (3)		6.4 (12)
13	.01	40	6.5 (?)	4.2 (3)	+1.9 (3)		6.1 (4)
Mean values						8.285 ±.045	6.285 ±.061

Note.—Runs 7, 11 and 13 and some not given above show uncertain indications of a third break at about 7.4 volts; G is the electronic current in amperes $\times 10^{-9}$.

V. DISCUSSION OF RESULTS.

Let us now compare the experimental results just presented with Lyman's data on the spectrum of nitrogen in the extreme ultra-violet given at the beginning of this paper. This is best done by writing corresponding values opposite each other in a table.

1. *The Break at* 8.29 *Volts.*—In this table we see that the effect which we got at all pressures tried and with various currents, that is the most

TABLE IV.

Spectrum Data.			Radiation Experiment Data.	
Remarks.	λ (Obs.).	V_0 (Calc.).	V_0 (Obs.).	λ (Calc.).
Doublet almost certainly due to nitrogen	1492.8 1494.8	8.28 8.27	8.29 ± .04	1490.7 ± 10
Doublet attributed to nitrogen but possibly due to silicon	1742.7 1745.3	7.08 7.09	7.3 (?)	1700 (?)
Beginning of band spectrum	1870.9	6.6	6.29 ± .06	1965 ± 20

intense effect, corresponds with greater accuracy than could be hoped for, to the most certain doublet of the line spectrum of nitrogen.

2. *Possible Effect at 7.3 Volts.*—The failure of the second value to coincide with that for the other doublet is no greater than the uncertainties of its determination. This radiation was apparently the weakest of the three and, while showing up well on one or two curves was, on the whole, rather doubtful. Attention should again be called to the doubt concerning the origin of this doublet as determined by Lyman.

3. *Explanation of 6.29-Volt Break.*—Two different theories were developed to account for this effect, one depending on the application of the quantum relation to the band spectrum and the other involving the idea of dissociation and the subsequent production of spectral lines. They are discussed at length in what follows.

(*a*) To explain the experimental result of a break occurring only at low pressures and coming at 6.29 volts instead of the 6.6 calculated from the band spectrum, we must consider the effect of a probable impurity. Kreusler[1] found that nitrogen prepared in a manner almost identical with that used in the present experiment had a small quantity of N_2O present as an impurity. He found that this increased absorption at $\lambda = 1{,}860$ from 2.2 per cent. for atmospheric nitrogen to 14.3 per cent. He also made measurements on pure nitrous oxide and found 88.4 per cent. absorption at $\lambda = 2{,}000$, apparently increasing beyond his powers of measurement at 1,930 and 1,860.

It is probable, therefore, that at the higher pressures, the radiation $\lambda = 1{,}965$ corresponding to 6.3 volts loses so much energy by absorption in the 1 cm. space between the gauze and the plate that it is not detectable. This explains the fact that there was no trace of this break at pressures above 0.026 and that it was only detected in one case at pressures exceeding 0.015.

[1] H. Kreusler, Ann. d. Phys., 6, p. 419, 1901.

The discrepancy between the value 6.3 found and the 6.6 corresponding to the beginning of the band spectrum is not so easily accounted for. It is a well known fact, however, that the presence of small impurities greatly affects the intensity of emission spectra. We have just seen that there is such an effect in the absorption spectrum of nitrogen in this region. Is it not possible then that the presence of N_2O may cause less refrangible bands up to $\lambda = 1{,}965$ to come out with sufficient strength to be detected? The chances for sufficient experimental error to account for a discrepancy of 0.3 volt do not seem great especially in view of the extremely good agreement in the case of the highest break. It is hoped that this point may be cleared up by further work taking every precaution to get absolutely pure nitrogen.

(*b*) The other explanation of our 6.3 volt break is quite different. There are lines in the spectrum of nitrogen in the region between 2,000 and 3,000 which would produce photoelectric effect and the most refrangible of which is $\lambda = 2{,}052$.[1] In the previous discussion it has been assumed that these arise from systems that are not present in this experiment, such as charged atoms or molecules, since otherwise we would have a photo-electric effect at lower voltages. If we allow the possibility of the production of some of these lines by a neutral atom, as we must for the doublet previously considered, then we must first have dissociation and then a radiating impact. Now the speed necessary for an electron to produce these radiations is between 4 and 6 volts, but the energy necessary to dissociate a nitrogen molecule is unknown. The velocity necessary for an electron to dissociate a hydrogen molecule is found by calculation from Langmuir's[2] results to be about 3.6 volts and it is known that nitrogen is much harder to dissociate. It is possible therefore that 6.3 is the speed necessary to dissociate the molecules and thus make it possible for the electrons of lower speed to produce radiation. This would correspond to a heat of dissociation of a gram molecule of nitrogen of 145,000 calories, whereas Langmuir[3] found for hydrogen the value 84,000.

This theory necessitates a new explanation of the effect of pressure, since these longer wave-length radiations will not be so strongly absorbed. It must be supposed that the chances for recombination of the atoms at higher pressures than 0.026 are so great as to make the radiation negligibly small.

As to the soundness of the assumption of radiation from neutral atoms, we have the following statement of J. J. Thomson regarding the lines of

[1] Lyman, "Spec. of Extreme Ultraviolet," p. 83, 1914.

[2] I. Langmuir, J. of Am. Chem. Soc., 37, pp. 417–458, 1915.

[3] Ibid., p. 457.

the hydrogen spectrum: "All theories concur in regarding the atom and not the molecule as the source of these lines, but according to Wien's theory the atom radiates when in the neutral state, while Stark maintains that the radiation is emitted when the atom has a positive charge: according to his view the lines emitted by the neutral atom are far away in the ultra-violet." [1] Stark[2] found no lines attributable to a neutral nitrogen atom but hardly carried his work below 4,000 A.U. From his work on mercury, however, he concluded that the line 2,536.7 was due to a neutral atom.[3] It is evident then that there is no evidence against our assumption but rather indications of its probability.

A second assumption implied in this theory is that the neutral molecules will not set up ultra-violet radiation when struck by electrons with speeds below 6.3. On this point, I have found no evidence.

4. *Upper Limit to Heat of Dissociation of Nitrogen.*—A necessary consequence of the principle that the line spectrum is due to atomic nitrogen is the determination of an upper limit for the energy necessary to dissociate a nitrogen molecule. If the effect resulting rom bombardment by electrons with velocities of 8.24 volts is due to atoms, the molecules must be dissociated by the impact of electrons of this or lower speed. The value 8.3 would give as an upper limit for the heat of dissociation of a gram molecule of nitrogen about 190,000 calories. As has been stated, Langmuir[4] found the value for hydrogen at constant volume to be 84,000 calories.

VI. IONIZATION AT 18.5 VOLTS.

Goucher and Davis found that what had previously been called ionization in nitrogen was really radiation but that true ionization did occur at 18.5 volts. Although the apparatus used in the present investigation was not well suited for testing this point, by greatly reducing its sensitivity a distinct increase in the slope of the electrometer current curve in the neighborhood of 18 volts was observed, thus supporting the more accurate work of Davis and Goucher.

It is realized that the present results were not obtained under experimental conditions ideal for getting sharp break points or reducing corrections to a minimum. The attempt was made, on the other hand, to employ experimental conditions in which the corrections would be as varied as possible, in order to test the soundness of the formula.

[1] J. J. Thomson, "Positive Rays," pp. 96–97, 1913.
[2] J. Stark, Ann. d. Phys., 55, pp. 29–74, p. 73, 1914.
[3] J. Stark, Ann. d. Phys., 52, pp. 241–302, p. 247, 1913.
[4] I. Langmuir, *loc. cit.*, p. 457, 1915.

Although the corrections in different tests differed by as much as 3 volts in extreme cases, the corrected values of the break point were quite consistent. It appears, therefore, that the above analysis is justified and necessary, and that the final values obtained are trustworthy.

The author wishes to express his indebtedness to Professor K. T. Compton, whose supervision and assistance have made this work possible.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.
July, 1919.

THE REFLECTION FACTORS OF TUNGSTEN AT INCANDESCENT TEMPERATURES.

BY W. WENIGER AND A. H. PFUND.

SYNOPSIS.

A direct method has been developed for measuring the reflection factors of substances at incandescent temperatures. Curves are given showing the variation in the reflection factor of tungsten for wave-lengths from 0.67 μ to 4.0 μ and for temperatures up to 2067° K.

The reflection factor of tungsten was found to increase with temperature for wave-lengths less than about 1.27 μ and to decrease with temperature for wave-lengths longer than this. The transition from the approximately constant positive temperature coefficient in the visible to the approximately constant negative coefficient in the infra-red takes place in the rather narrow region lying between about 0.7 μ and 2.0 μ. The numerical values of the reflection factor agree fairly well with Worthton's values in the visible and with the theoretical equation first applied to long wave-lengths by Hagen and Rubens, beyond 2 μ in the infra-red.

THIS study of the reflection factors of tungsten was made as part of the general study of the properties of tungsten undertaken by Nela Research Laboratory. Closely related papers on the emissive powers in the visible and ultra-violet have already been published by Worthing[1] and Hulburt[2] respectively. The present paper deals with the reflection factors at incandescent temperatures in the red end of the visible spectrum and the near infra-red. The most direct method of attack was adopted, involving spectral energy measurements of a beam reflected from a polished tungsten surface both when hot and when cold.

APPARATUS AND METHOD.

The success of the method depended entirely on obtaining a tungsten mirror that could be heated without appreciable warping. A few trials were made with massive mirrors, including an attempt to heat such a mirror by means of an electronic discharge impinging upon its back; but it was found simpler and entirely practicable to use smaller mirrors that could be heated by means of an electric current passing through them. Three different types of such mirrors were made: (*a*) squirted filament, circular hole along axis, outer surface hexagonal, width of face 0.75 mm.; (*b*) swaged filament 1.16 mm. in diameter, central portion

[1] PHYS. REV., 10, 1917, p. 377.

[2] Astrophys. Jour., 45, 1917, p. 149.

rolled flat and bent into a form resembling that of the old-fashioned paper fasteners known to the trade as "flat heads," mirror face 25 x 2 mm.; (*c*) drawn wire, 0.9 mm. in diameter, bent so as to form three sides of a long narrow rectangle, ground flat so that its section became a semicircle. All were heated in vacuo to a temperature higher than that used in the experiments, were rough-polished on cloth with sulphur-flour and rouge, and finished on a pitch surface with the finest rouge; and were mounted in evacuated glass bulbs. The part of the polished surface used was always sufficiently removed from the lead-in wires so that its temperature was uniform. The data taken on mirrors *a* and *b* were considered preliminary; those on *c* are recorded below. All mirrors showed the same general behavior. In this connection two experimental difficulties should be mentioned: (1) it was very difficult to make stem seals that would not check, due to the frequent changes in the large heating currents; (2) it was found that the mirror surface developed furrows, which, however, obliterated only a small percentage of the surface, the parts between the furrows retaining their polish.

The spectrobolometer was of the Wadsworth fixed arm type; instrumental dimensions are given in the legend accompanying Fig. 1. The bolometer and galvanometer were provided with auxiliary apparatus so that the galvanometer could be used either as a deflection or as a zero

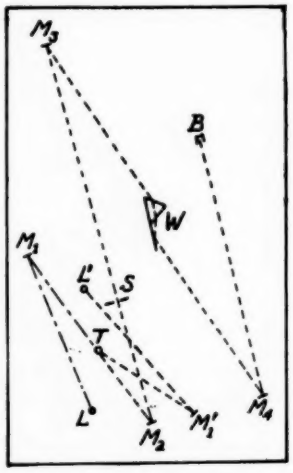


Fig. 1.

Diagram of Apparatus.

L, *L'*, two positions of flat filament (0.2 × 2.5 cm.) tungsten lamp in cylindrical bulb, operated at 15 amp.

M_1, M_1', two positions of concave mirror, $f = 20.0$ cm., diam. = 9.3 cm. *L* and *M* were mounted on a single tripod which could be placed in either one of two sets of hole-slot-plane plates. The primed position served for measurements of the beam reflected from *T*, and the unprimed for measurements of the beam incident on *T*.

T, tungsten mirror, mounted as a lamp filament in a selected tubular glass bulb; see text. Angle of incidence of beam on mirror 10°.

M_2, concave mirror, $f = 16.0$ cm., diam. = 15 cm.

S, slit, 0.5 mm. × 20 mm. Subtends 0.13 μ in the most condensed part of the spectrum near 1.63 μ.

M_3, M_4, concave mirrors, $f = 76$ cm., diam. = 15 cm.

W, fluorite prism in Wadsworth mounting. Prism angle 59° 59.8'.

B, bolometer, platinum strips (0.5 × 25.0 mm.) Smoked above acetylene flame. Resistances when total bolometer current is 0.12 amp., are 5.303 and 5.296 ohms. Balance coils 10 ohms each, part of Wolff five dial bridge. Galvanometers: Four-coil, astatic, 7-sec. period, volt sensitivity, 670 mm. per microvolt at a scale distance of 1 meter. *S-H* d'Arsonval, critically damped, volt sensitivity, 10 mm. per microvolt at a scale distance of 1 meter. Galvanometer scale actually used at distances of 2.5 m. from the four-coil instrument and 5 m. from the d'Arsonval.

instrument. When used as a deflection instrument, its sensibility could be reduced by a group of shunts, but when used as a zero instrument it was always kept at full sensibility. For use in the latter manner, the exposed strip of the bolometer was shunted by a high variable resistance (10,000 Ω in parallel with the strip and 3,000 of this 10,000 in parallel with from 1 to 110,000 Ω) so that at a balance with the exposed strip illuminated, the current in each of the four arms of the bridge was the same as when the shunt was disconnected and the strip shaded. The changes in current due to exposures of the strip, as is customary, were assumed proportional to the incident radiant flux and their values were calculated from the experimentally determined values of the shunt resistance necessary to bring this current back to its normal value.

The path of the beam before reaching the slit of the spectrometer is indicated in Fig. 1 by $L'M_1'TM_2S$. To insure that the tungsten mirror T was in exactly the same position in the image of the source formed by M', when hot as when cold, two telescopes were mounted permanently, one near M_2 giving a view of the front of the mirror, and the other off at right angles, giving a view of the edge of the mirror, the cross hairs of both being set on marks that could be easily identified. A shutter was placed near M_1'. The natural radiation from the tungsten mirror was allowed to enter the slit at all times and the increase in the galvanometer deflection due to opening the shutter was recorded. Readings were usually made in this order: mirror at room temperature, mirror incandescent, mirror at room temperature.

The actual galvanometer deflections or resistance readings varied in magnitude from day to day, due to using different currents in the bolometer and in the source, etc., and are, therefore, not recorded below. Between 0.7 μ and 3.0 μ no deflections less than 2 cm. nor greater than 20 cm., were used; readings were made to 0.1 mm.; each figure recorded is based on the mean of at least ten deflections. Spectrometer settings were checked at the beginning and end of each run by the yellow and red lines of helium and occasionally by the carbon dioxide emission band at 4.4 μ. The temperature-current relations for the tungsten mirrors were determined with a Holborn Kurlbaum optical pyrometer by Dr. Worthing of this laboratory.

DATA AND DISCUSSION.

The data are recorded in Table I. and plotted in Figs. 2 and 3. In a narrow region bounded approximately by the wave-lengths 0.7 μ and 2.0 μ a change of temperature produces a striking change in the relation between reflection factor and wave-length. Within this interval an

TABLE I.

Relative Reflection Factors of Tungsten,

i.e., $\frac{\text{Reflection factor at high temperature}}{\text{Reflection factor at room temperature}}$.

Wave-length in μ.	1380° K.	1632° K.	1859° K.	2067° K.
0.67	1.06	1.074	1.087	1.098
0.70				1.090
0.76				1.089
0.82				1.080
0.90				1.065
1.05				1.040
1.10				1.026
1.16				1.014
1.20				1.012
1.27	1.000	1.000	1.000	1.000
1.39				0.976
1.50				.954
1.63				.928
1.76				.909
1.90	0.933	0.919	0.902	.890
2.00	.925	.908	.891	.877
2.23				.876
2.44				.876
2.90	.923	.906	.889	.876
2.99				.878
4.00				.880

increase in temperature causes an increase in the reflection factor for wave-lengths less than about 1.27 μ and a decrease for wave-lengths

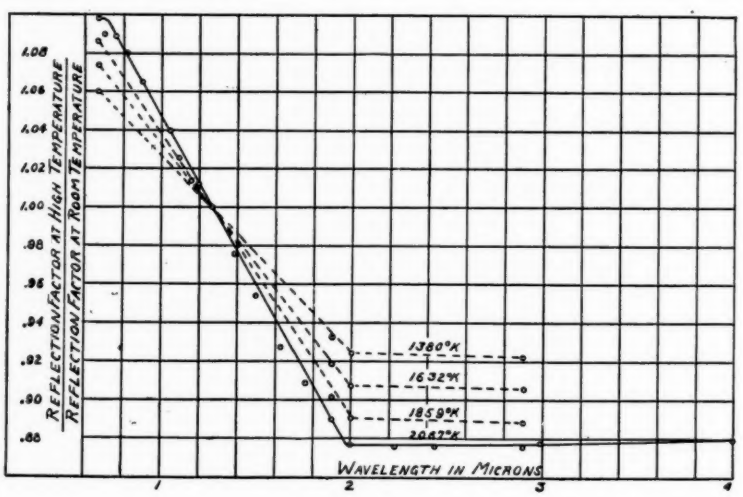


Fig. 2.

Relative Reflection Factor.

greater than this, the change in each case being greater the higher the temperature.

Previous experimental work has shown that in the visible part of the spectrum, the reflection factor of tungsten increases with temperature; thus Worthing[1] for wave-length 0.665 μ using an optical pyrometer method which for this region of the spectrum is much more delicate than that employed by the present authors, gives data yielding a change in

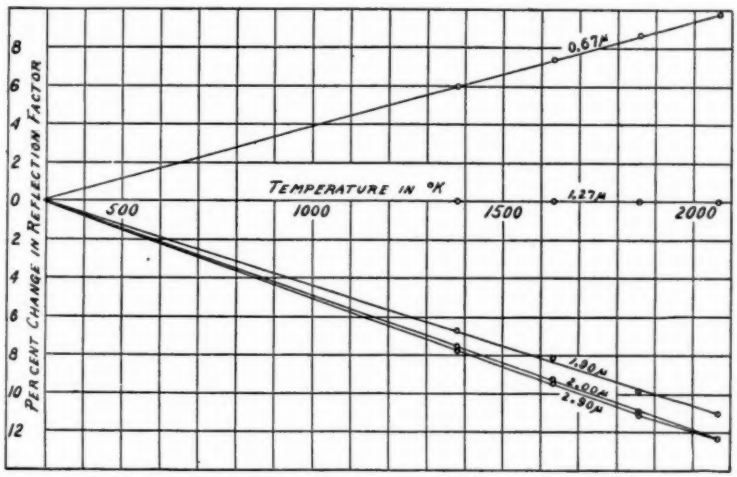


Fig. 3.

Change in Reflection Factor with Temperature.

the reflection factor of 5.0, 6.2, 7.3, and 8.2 per cent. at temperatures of 1380, 1632, 1859, and 2067° K. respectively; the corresponding values found here are 6.0, 7.5, 8.7, and 9.9 per cent.

As is well known, Hagen and Rubens,[2] showed that for long wave-lengths the reflection factors of metals obey the formula derived by Drude,[3] from Maxwell's theory:

$$R = 100 - 3650\sqrt{\frac{\rho}{\lambda}}$$

R is the reflection factor in per cent., ρ is the resistivity in ohm-cm., and λ the wave-length in μ. That this formula fits the present data fairly well beyond 2.0 μ may be seen from Table II.

The resistivities are taken from Langmuir's table.[4] The values of "R Observed" at room temperature (293° K.) are taken from Coblentz'[5]

[1] PHYS. REV., 10, 389, 1917.

[2] Ann. d. Phys., 4F, 11, 873, 1903; Phys. Zeitschr., 11, 139, 1910.

[3] Lehrbuch der Optik, 2d ed., p. 349.

[4] PHYS. REV., 7, 154, 1916.

[5] B.B.S., 14, 312, 1918.

TABLE II.

Comparison of Observed and Theoretical Values of the Reflection Factor of Tungsten for long Wave-Lengths.

Temperature.	Resistivity Ohm-Cm.	Change in R (Fig. 2).	R Observed.	R Computed.	$\frac{R \text{ Computed}}{R \text{ Observed}}$.
		$\lambda = 2.0\,\mu$			
293° K.	5.51×10^{-6}		90.0%	94.0%	1.045
1380	37.46	− 7.5%	83.3	84.2	1.011
1632	46.01	9.2	81.8	82.5	1.009
1859	54.19	10.9	80.3	81.0	1.009
2067	61.99	−12.3	79.0	79.7	1.009
		$\lambda = 2.5\,\mu$			
293° K.	5.51×10^{-6}		93.8%	94.6%	1.009
1380	37.46	− 7.6%	86.6	85.9	.992
1632	46.01	9.3	85.0	84.3	.992
1859	54.19	11.0	83.5	83.0	.994
2067	61.99	−12.3	82.3	81.8	.994

most recent table; the rest of the numbers recorded in these columns are the result of applying to these values the percentage changes determined by the present investigation. The columns headed "R Computed" are obtained directly from the formula written above. The approximate constancy of the ratio of the computed to the observed values, which holds also at $3\,\mu$, indicates that the formula holds for tungsten at considerably shorter wave-lengths than for other metals previously studied. In fact it appears that the formula holds up to the point at which the sharp break occurs in the curves in Fig. 3, that is up to the region in which the rapid transition takes place from a decrease to an increase in the reflection factor with an increase of temperature.

The general character of the changes found in the reflection factor of tungsten are not peculiar to this metal; McCauley[1] using an indirect method based on Kirchhoff's law, $E/\epsilon = 1 - R$, in which he measured E, the radiant flux emitted by the substance at a certain temperature and wave-length and obtained ϵ, the flux emitted by a black body at the same temperature and wave-length, from Planck's equation, found similar changes in the reflection factors of platinum, palladium and tantalum.

NOTE ON THE REFLECTION FACTOR AT ROOM TEMPERATURE.

The authors measured the reflection factor of tungsten at room temperature by a direct method with the apparatus in front of the slit

[1] Astrophys. Jour., 37, 164, 1913.

arranged as indicated in Fig. 1, so that the transition from a measurement of the incident beam to that of the reflected beam could be made readily, with the two optical paths identical except for the reflection from the tungsten mirror.

A curve showing the relation between reflection factor and the wavelength plotted from data taken after the conclusion of the high temperature experiments differs markedly from a similar curve taken when the mirror was new. Not only does the earlier curve show slightly higher values of the reflection factor, as might be expected on account of the fissures that appeared in the surface during use, but the earlier curve also shows the two depressions at about 0.8 μ and 1.4 μ shown by Coblentz,[1] whereas these seem to be absent in the later curve. As the actual behavior of the metal throughout the infra-red and particularly in this region of the spectrum is of considerable interest, a further study, using this direct method and greater dispersion, is contemplated.

NELA RESEARCH LABORATORY,
CLEVELAND, OHIO,
June 28, 1919.

[1] *Loc. cit.*

REFLECTION OF ELECTRONS FROM METAL SURFACES.

BY H. M. DADOURIAN.

SYNOPSIS.

A beam of electrons from a hot-lime cathode was introduced into the space between two parallel plates so as to be incident against one of them at a large angle. Then the velocity distribution among the resulting secondary electrons was studied by observing the number of electrons which, overcoming different retarding potentials reached a Faraday cylinder. The experimental results obtained tend to show that the number of secondary electrons having velocities between zero and a given velocity v may be expressed as the sum of two numbers one of which is proportional to the number of electrons having the velocity v and the other of which is proportional to the energy of X-rays in the space between the two plates having the wave length which according to the quantum theory corresponds to the velocity v.

THE following investigation was undertaken in order to study the velocity distribution among the secondary electrons produced by the incidence of a beam of cathode rays against a metal surface, and to determine the relation between the velocity of impact against a metal target and the velocity of reflection of an electron.

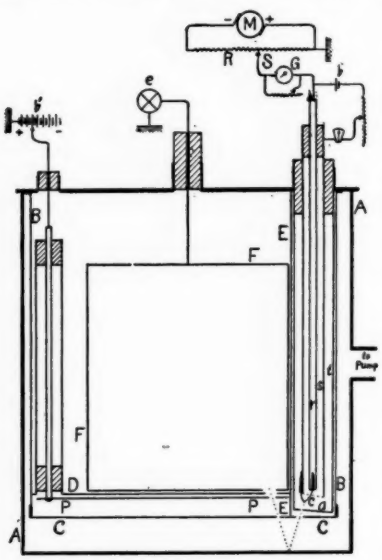


Fig. 1.

A sketch of the apparatus used is shown in Fig. 1, where AA is a copper cylinder about 4 inches in diameter; BB is a brass cylinder provided with a transverse partition D, a longitudinal partition EE, and a detachable cap CC; FF is a Faraday cylinder, carefully insulated and connected to an electrometer of the Doletzalek type; c represents a hot-lime cathode which consists of a narrow strip of platinum connected at one end to the rod r and at the other end to the tube s.

The platinum strip had a small patch of barium deposited upon it by burning away a tiny speck of Bank of England sealing wax. The heating current of the cathode was supplied by a battery, b, of low voltage,

while the cathode potential was furnished by a motor generator, *M*, the terminals of which were connected to the ends of a graphite rod, *R*, which had a resistance of about 95,000 ohms. At full speed the generator maintained a fairly constant potential of about 1,500 volts between the ends of the graphite rod; consequently any desired potential up to 1,500 volts could be applied to the cathode by moving the slide-contacts along the graphite rod. The constancy of the current of cathode electrons from the hot-lime cathode to the anticathode was observed by means of the galvanometer *G*.

When the apparatus is in operation a beam of cathode rays passes through a hole in the base *a* of the tube *t*, which forms the anticathode, and produces secondary electrons by striking the brass bottom of the cylinder *AA*. Some of these electrons pass through the holes in *CC*, *PP*, *D* and *FF* into the Faraday cylinder and cause the electrometer needle to deflect.

The holes in *a*, *CC* and *PP* were, respectively, 0.5 mm., 1.0 mm. and 3.0 mm. in diameter. The right-hand hole in *CC* and those in *D* and *FF* were large enough not to affect the passage of electrons. The distances from the bottom of the apparatus of the center of the source of electrons on the hot-lime cathode, of the base *a*, of the cap *CC* and of the plate *PP* were, respectively, 19 mm., 13 mm., 10 mm. and 18 mm. The centers of the holes and of the source of electrons were in the same vertical plane. The distances of the centers of the holes in *a*, *CC* and *PP* from the vertical line through the center of the source of electrons were 2 mm., 12 mm. and 14 mm., respectively.

The distribution of velocity among the secondary electrons was studied by applying different retarding potentials to the plate *PP* and observing the corresponding rate of deflection of the electrometer. This rate of deflection was obtained by taking the time of passage of an image reflected from the electrometer mirror over 20 cm. of a scale placed at a distance of one meter from the electrometer. The image was allowed to pass over 5 cm. of the scale before measurement of time was begun.

The results of some of the experiments are indicated by the curves of Figs. 2 to 5, where the abscissas denote the retarding potentials applied to the plate *PP* and the ordinates the rate of deflection of the electrometer. The potential applied to the hot-lime cathode during each experiment is indicated in the figures. The curves of Fig. 2 and Fig. 3 were obtained with the cap *CC* on, and those of Fig. 4 and Fig. 5 with it off.

It will be observed that the greater part of each of the curves has the general appearance of an exponential curve. Let us suppose for the

moment that the experimental curves are true exponential curves represented by the equation

$$I = I_0 e^{-\lambda V}, \tag{1}$$

where λ is a constant, I_0 represents the number of electrons which enter the Faraday cylinder when no retarding potential is applied, and I the

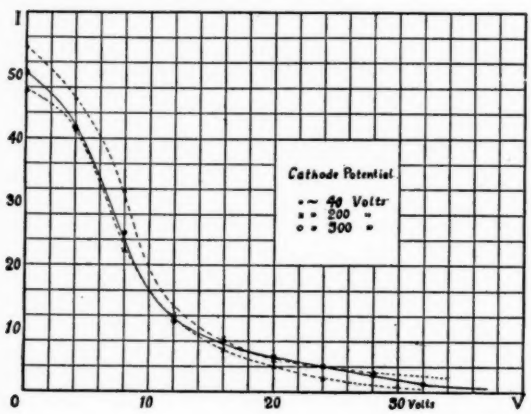


Fig. 2.

number when a retarding potential V is applied to the plate PP. The number of electrons which are prevented by the retarding potential from entering the Faraday cylinder is then given by

$$I_0 - I = I_0(1 - e^{-\lambda V}). \tag{2}$$

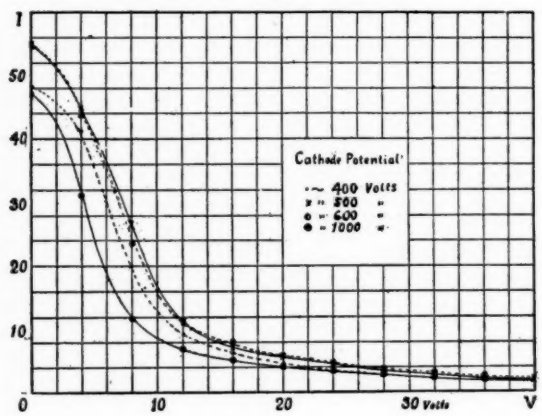


Fig. 3.

Therefore the number, n, of electrons having the energy corresponding to a potential V is obtained by differentiating equation (2)

$$n = -\frac{dI}{dV}$$
$$= \lambda I, \tag{3}$$

or

$$I = \frac{\mathrm{I}}{\lambda} n. \tag{4}$$

In other words the total number of electrons which enter the Faraday

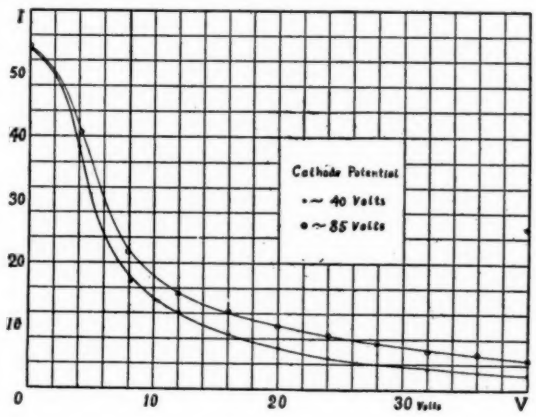


Fig. 4.

cylinder having velocities between zero velocity and that corresponding to a potential V is proportional to the number of electrons having the higher velocity.

The curves differ, however, from true exponential curves in two

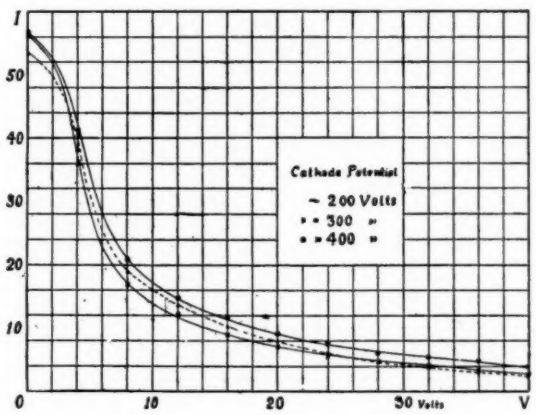


Fig. 5.

respects. First, in nearly all cases there is a point of inflection near the I-axis. Secondly, beyond the inflection point the ordinates decrease more rapidly for smaller abscissas than would be expected if the curves were represented by equation (1). The first may be due to errors in

measuring the interval of time taken by the image from the electrometer mirror to cover 20 cm. of the scale. For zero retarding potential this interval of time amounted to between two and three seconds; consequently it could not be determined very accurately by the stop watch used for the purpose. The change in curvature may, on the other hand, be real, in which case the velocity distribution curve obtained by differentiating the current-potential curve has a maximum point.

The second difference may be accounted for by supposing that the current-potential curves are the resultants of two exponential curves. In fact a very close approximation to the experimental curves is obtained by putting

$$I = I_1 + I_2, \tag{5}$$

where

$$I_1 = I_0' e^{-\lambda' V} \tag{6}$$

and

$$I_2 = I_0'' e^{-\lambda'' V}. \tag{7}$$

On this assumption the total number of electrons, I, entering the Faraday cylinder and having velocities from zero up to that corresponding to the potential V equals the sum of two numbers, each of which is proportional to the amount of a different type of energy in the space between the bottom of the cylinder AA and the cap CC (Fig. 1). The meaning of this statement will become clearer when we consider the transformation of energy which takes place in this space. Energy is admitted into this space in the form of energy of motion of cathode electrons moving with a velocity corresponding to the cathode potential. This energy changes progressively from energy of electrons into energy of soft X-rays and vice versa. In the process of this transformation energy is degraded to the lower forms represented by electrons of slower velocities and softer X-rays. The total number of electrons having velocities between zero and that corresponding to any potential V is then equal to the sum of two numbers one of which is proportional to the number of electrons having velocities given by the expression

$$\tfrac{1}{2}mv^2 = Ve, \tag{8}$$

where m, e and v are respectively the mass, the charge and the velocity of the electrons; while the other number is proportional to the amount of energy of X-rays of a frequency given by the quantum relation

$$h\nu = Ve, \tag{9}$$

where h is Planck's constant and ν is the frequency of the rays.

While working on an investigation on total ionization[1] J. B. Johnson

[1] J. B. Johnson, PHYS. REV., 10, 609, 1917.

obtained certain results which the present writer interpreted as being due to multiple reflection of electrons between two metal plates, and on this hypothesis he computed the velocity of reflection and found it to be two-thirds of that of the velocity of impact of the electrons. Mr. Johnson made an attempt at the time to repeat his results, but was unable to do so, and being pressed for time turned his attention to the problem of total ionization. It was thought worth while, therefore, to try to obtain results which would agree with those which Johnson could not repeat and which would lead to the determination of the relation between the velocities of impact and of reflection of an electron.

If the velocity distribution of electrons in the space between the bottom of the apparatus of Fig. 1 and the cap *CC* were due primarily to multiple reflections from the two surfaces the ordinates of the curves of Figs. 2 to 5 would have decreased in steps as the retarding potential was increased. Then the relation between the velocities of impact and of reflection could be computed from the potentials at which the sudden drops in ordinates occurred. The absence of sudden drops in the ordinates of the experimental curves obtained in these experiments indicates that the velocity distribution is due to several factors, such as suggested in the preceding paragraph, which tend to make the velocity distribution curves continuous.

In conclusion the author wishes to thank Professor George B. Pegram, of Columbia University, for extending to him the facilities of the Phœnix Physical Laboratory where this investigation was carried out, and also Professor Bergen Davis for helping him to secure the apparatus necessary for the experiments and for many other courtesies which made the author feel at home in a strange laboratory.

Trinity College,
Hartford, Conn.,
August, 1919.

REMARKS ON DR. L. SILBERSTEIN'S RESULTS IN HIS PAPER "ON THE DISPERSION OF THE DIAMOND."[1]

BY ALBERT C. CREHORE, PH.D.

SYNOPSIS.

Dr. Silberstein begins with the assumption that dispersion in general is due to the interaction of the electromagnetic field with the electrical charges on the nuclei of atoms. In the particular case of diamond he deduces from this assumption that the charge on the nucleus of the carbon atom is 2.22 times the electronic charge. He admits that this result is not in harmony with the pan-electronists belief in the indivisibility of the electron. This conclusion justifies us in bringing into question his fundamental premises. It is the author's belief that spectrum effects are in general principally due to the revolving electrons rather than the nuclei of atoms. Reasons for this are given in a book by the author now in press. In the case of diamond, if the Silbertsein ratio, 2.22, which is corrected to read 2.246, is multiplied by the characteristic number of the tetrahedron, namely 8/3ds, the result becomes 5.99, which is very close to the whole number, 6, the atomic number of carbon. The 8/3ds would be required if the effect is due to the revolving electrons. It is pointed out that the dimensions of equations derived from electromagnetic theory by most modern writers are not correct without expressing the specific inductive capacity of the medium. In particular this point is illustrated by Dr. Silberstein's equation (9), and the subject is fully treated in the author's book referred to.

IN a paper "On the Dispersion of the Diamond," Dr. Silberstein assumes that the dispersion of the diamond is due to the action of the nuclei of the carbon atoms, and he proceeds to derive the so-called "atomic coefficient" of carbon on this assumption. In the opening paragraphs he says: "Consider a space-lattice of points, indefinitely extended in all directions. Let all these points be occupied by the centers of *equal* atoms, each containing a single electron whose charge and mass are e, m."

The conclusion[2] arrived at as a result of this premise makes "*the atomic coefficient of carbon*, entering into its

$$\text{'atomic refractivity'}\ C = \frac{k_0}{u_0 - u},"$$

$$k_0 = 4.07 \times 10^{10}\ \text{cm. gr.}^{-1}.$$

The numerical value of k_0, which is called ϵ, when attributed to a single electron placed at each point of the space-lattice, as given on page 404, is

$$\epsilon = 1.83 \times 10^{10}\ \text{cm. gr.}^{-1}.$$

[1] L. Silberstein, Phil. Mag., Vol. XXXVII., No. 220, April, 1919, p 396.

[2] *Loc. cit.*, p. 405.

The quotient of k_0 above obtained from the diamond, and supposed to be due to the carbon atom, by this ϵ is about 2.22, giving the numerical ratio

$$k_0 \text{ (for diamond)} = 2.22\,\epsilon.$$

Dr. Silberstein remarks on this result: "This exceeds the nearest multiple ($2\,\epsilon$) of the proper electronic value by as much as $0.22\,\epsilon$, a feature certainly not agreeable to the pan-electronists. I do not propose here to attempt to bring it down to an exact multiple by some artificial assumptions."

Believing that the pan-electronists have a very strong case, it seems necessary to question the conclusion of Dr. Silberstein. Assuming that his deductions are all correct on the premises, this is equivalent to doubting the truth of his assumptions.

Briefly, it is my belief that the discrepancy between the ratio 2.22 obtained by Dr. Silberstein and a whole number is due to a cause so fundamental that it negatives the original premises that the dispersion of the diamond is due to the nuclei of the atoms at all. It is my belief that the dispersion is due to the revolving negative electrons rather than the positive nuclei, and further that all spectrum effects including both light and x-ray spectra are due primarily to the motion of the revolving electrons when displaced out of their normal circular orbits. The reasons for this belief are given in a book[1] by the author now in press.

In the present case these views are supported by the result of Dr. Silberstein, for, if we multiply the ratio that he obtains, namely 2.22 above mentioned, by the numeric 8/3ds, a value close to a whole number, 6, is obtained, which might be expected for the diamond, this being the atomic number of carbon. Now, this 8/3ds is a characteristic number relating to the regular tetrahedron, as has been pointed out in several papers.[2]

If we go back and alter the assumption made at the beginning by Dr. Silberstein, and say that in place of the single charge that he assumes to be at each point of the lattice, thus making equal atoms, we now require a group of four revolving electrons, revolving in orbits inclined to each other as the four medial lines of a regular tetrahedron, the result will introduce the required factor 8/3ds. It is not supposed, however, that four such electrons exist in a single atom, but that four atomic centers are required to furnish them, one electron from each atom; for, it has been shown that the directions of the axes of revolution of the

[1] A. C. Crehore, "A New Theory of the Atom," D. Van Nostrand Co., New York, 1919.

[2] Crehore, PHYS. REV., Vol. IX., No. 6, Sec. Ser., June, 1917, pp. 458–460; Vol. XII., No. 1, Sec. Ser., July, 1918, pp. 18–21; *loc. cit.*, A New Theory of the Atom, Appendix A.

electrons in the atoms of the diamond are such that the whole diamond may be divided into four equal groups of atoms,[1] each group having the direction of its axis of revolution parallel to one of the medial lines of the regular tetrahedron.

To compare the k_0, which is called the atomic coefficient, however, with the effect of a single stationary electron, one at each point of the space-lattice, we may retain the same expression for k_0, that is for ϵ, that Dr. Silberstein uses, namely,

$$k_0 = \frac{1.008}{3} \times \frac{e'/c}{m_0} \times \frac{e'/c}{m_H}, \tag{10}$$

where e' denotes the charge on one electron proper. The numerical value of this expression as given by him on page 404 is $\epsilon = 1.83 \times 10^{10}$ cm. gr.$^{-1}$. The numerical values of e', m_o and m_H used by him to obtain this value are not stated. Let us, first, recalculate this numerical value making use of the theoretical values of e, m_0 and m_H as given in my book[2] referred to, chapter X.

$$e = 4.763 \times 10^{-10},$$
$$m_H = 1.658 \times 10^{-24},$$
$$m_0 = 0.898 \times 10^{-27}.$$

The even number, 3×10^{10}, has been employed throughout for the velocity of light in deriving these theoretical numerical values of e, m_0 and m_H, and will be employed again here because the true value of the velocity of light is very close to and a little less than this even number, and authorities apparently differ as to its more exact value. The Smithsonian Tables give its value due to Weinberg as $c = 2.99870 \times 10^{10}$, and I believe that Millikan has used a value $c = 2.999 \times 10^{10}$ which is the same if the last two digits above are omitted. This differs from the even number 3 by only one unit in the fourth significant figure. With these values we obtain for the Silberstein, ϵ,

$$\epsilon = 1.8107 \times 10^{10}, \text{ instead of } 1.83 \times 10^{10} \text{ above.}$$

Let us next recalculate the Silberstein k_0 for the diamond using his formula

$$k_0 = \frac{1}{3} \frac{M}{d} a,$$

[1] Crehore, Phil. Mag., Vol. XXIX., June, 1915, pp. 763, 766–767; Vol. XXX., Aug., 1915, p. 257, "Construction of the Diamond with Theoretical Carbon Atoms."

[2] The book referred to has been delayed because of the printer's strike. These numerical values of e, m_H and m_0 are also derived in Section III. of a series of articles entitled "The New Physics," the contents of which was published in The Journal of Electricity, San Francisco, Cal., October, 15, 1919, which begins the series.

and the same numerical values as he gives, namely

$M = 12$, atomic weight of carbon,

$d = 3.515$, mean density of diamond,

$a = 3.5740 \times 10^{10}$, obtained from Marten's observations on the diamond and the Silberstein formula.

We find

$$k_0 = 4.06714 \times 10^{10}.$$

The decimal places are retained here merely on the supposition that the numbers from which this is derived are correct so far as given, the error being unknown.

Introducing, now, the new hypothesis that the dispersion is wholly due to the revolving electrons and not to the nuclei of the atoms, it is necessary to embrace four atoms in one fundamental unit before we have a right to assume that the dispersive centers are all equal. Such groups will be equal, but not the individual atoms because of the orientation of their axes with respect to each other, which must be taken into the account. Each electron on the average in the tetrahedron formation is just 2/3ds as effective[1] as it would be without any orientation. If, therefore, we multiply the Silberstein formula above given by 2/3ds on account of the reduced effectiveness of the electrons due to the tetrahedral formation, and also multiply the value of M by four so as to embrace four atoms in a unit instead of a single one, the formula becomes

$$k_0 = \frac{8}{3}\frac{1}{3}\frac{M}{d}a = \frac{8}{3} \times 4.06714 \times 10^{10} = 10.8457 \times 10^{10}$$

for the diamond. Taking the ratio of this coefficient to the value of ϵ above obtained on the single electron hypothesis, namely 1.8107×10^{10}, the ratio is obtained as follows:

$$\text{Ratio} = \frac{10.8457}{1.8107} = 5.99.$$

The ratio obtained by Dr. Silberstein as above stated, assuming the dispersion to be due to the nuclei, was 2.22, which led him to make the remark above quoted, thus bringing into question a fairly well established fact, namely, the indivisibility of the electron. The new ratio just obtained is very close indeed to the whole number, 6, that is, to the atomic number of carbon, which is supposed to be 6. The difference between 5.99 and 6.00 may easily be accounted for by an error in the assumed mean density of the diamond without assuming that any error at all has entered through the other constants. Dr. Silberstein remarks in a footnote, page 405, that "Martens does not quote the density of his

[1] *Loc. cit.*

piece of diamond." The required density for the diamond, which will make this ratio exactly 6 is

$$d = 3.509,$$

which differs by only six units in the fourth digit from the value adopted by Silberstein.

In submitting these remarks on the Silberstein theory there is no claim to have produced a complete new theory on the new hypothesis that the dispersion is due primarily to the revolving electrons with any such thoroughness as Dr. Silberstein has developed his theory. The fact, however, that the factor 8/3ds reduces his ratio with such exactness to the whole number, 6, which is also the atomic number of carbon, is of considerable interest in itself. The coincidence is too near to be merely the result of chance.

In conclusion I cannot refrain from referring to the dimensions of the equations used by Dr. Silberstein in this paper in common with other writers on electromagnetic theory, because we have another example here of the common practise of omitting to write the specific inductive capacity in all equations derived from electromagnetic theory. Referring to the equation (9) page 403, namely

$$\mu^2 = 1 + \frac{a}{u_0 - u}, \tag{9}$$

the dimensions of each term should be the same, namely the same as μ^2, that is, dimensionless in terms of length and of time. For, index of refraction is essentially a ratio between two sines or two velocities, and is, therefore, dimensionless in terms of any fundamental units. The numeric, 1, in the second member satisfies this requirement, but the last term, $a/u_0 - u$, does not satisfy it. The value of a, as given in equation (8), is

$$a = \frac{NB}{4\pi^2 c^2} = \frac{Ne^2}{4\pi^2 c^2 m}.$$

The meaning of N is the number of carbon atoms per unit volume, and its dimensions may be taken as $N = L^{-3}$. The dimensions of e^2 on the electrostatic system of units are $L^3MT^{-2}k$, the k being the specific inductive capacity of the medium. Dividing e^2 by c^2m gives the dimensions of e^2/c^2m as Lk. Multiplying by the dimensions of N gives for the dimensions of his, a, $L^{-2}k$.

The dimensions of $u = 1/\lambda^2$ are simply L^{-2}, and not $L^{-2}k$, which are the dimensions of a. The quotient $a/u_0 - u$, therefore, has the dimension, k, and is not dimensionless like the other terms in the equation.

Of course, the numerical value of the expression is not affected at all

by the omission of the k so long as the second and the centimeter are used as our units of time and of space, but, if these units should be changed, then the k is no longer unity, and it does make a great difference in the numerical value. This matter is explained at some length in the book above referred to, and it does not seem necessary to enlarge upon this point further here.

This last criticism on the matter of dimensions is not at all peculiar to Dr. Silberstein's paper, but applies to most writers on electromagnetic theory at the present time. And, indeed, no criticism whatever has been made of Dr. Silberstein's work so far as deductions from his premises are concerned. His conclusions, however, seem to lead him to question what many consider to be a well-established fact, and the object of these remarks is to discover a way out of this difficulty.

AN EXPERIMENTAL METHOD FOR THE PRODUCTION OF VIBRATIONS.

BY C. V. RAMAN.

SYNOPSIS.

Forced Vibration of strings; the Melde experiment. Improvements of Fleming's modification are described which enable the amplitude of the forced vibration to be varied, which minimize lateral vibration, and which make it possible to obtain linear, circular or elliptical vibration as desired. This improved apparatus is useful for studying or demonstrating the various types of motion possible for a stretched string. Five photographs illustrate the results which may be obtained.

Forced Vibration of strings. The phenomenon observed by Jones and Phelps, that when two or more different types of vibration are simultaneously obtained, the ratio of the number of ventrals egments may differ from the ratio of the frequencies of maintenance, is briefly discussed.

PROF. J. A. FLEMING[1] has recently devised and described a very simple but ingenious and compact piece of apparatus to illustrate the close analogy existing between the transmission of mechanical vibrations along a loaded string and the propagation of alternating electric currents along a telephone cable having inductance coils inserted in it at equidistant intervals. For this purpose, the well-known method of producing forced vibrations on strings due to F. Melde is not suitable; for, when a long heavily loaded string is used, very large forks would be required, and the reaction of the string when under tension would generally result in stopping the fork even if the latter be electrically driven. Further, it is also desirable to have a means of altering the frequency of vibration which is not possible with Melde's method except by changing the fork. Prof. Fleming therefore replaced the fork in the Melde experiment by a small continuous-current electric motor, on the shaft of which a disk carrying an excentrically placed pin is fixed. The pin actuates ă light crank shaft, which is connected at the other end to a rocking lever. The string is maintained in vibration by having its extremity attached to a small hook fixed to the crank-shaft. With an arrangement of this kind, it is possible to impose either a linear or circular vibratory movement on the extremity of the string as desired.

In experimenting with an apparatus of the type described above, the present author found that its usefulness is considerably enhanced by

[1] Physical Society, London, Proc., December, 1913, p. 61.

certain improvements which he has made in it. In Fleming's apparatus, as illustrated in his paper, the excentric pin is permanently fixed to the disk carried on the shaft of the motor. The amplitude of the obligatory circular or linear vibration imposed on the extremity of the string is consequently invariable. This is a disadvantage, as in the practical use of the apparatus, it is often useful to be able to alter the amplitude of the obligatory motion quickly to any value, either large or small, as may be desired. Further, the author has found that with an apparatus of the Fleming type, when the shaft of the motor is in rapid rotation, the rocking lever and crank-shaft tend to develop vibrations in a direction transverse to their plane, and this introduces undesirable complications in the forced vibration of the extremity of the string. To remedy these defects and make the apparatus a real precision instrument for the study of vibrations, the design of the working parts has been modified as shown in Fig. 1. The disk fixed to the shaft of the motor has a slotted plate screwed to it a little above the general surface, and the crank-pin is carried by a sliding piece which can be put at any desired point and fixed by a single half-turn of the screw-head shown in the figure. The rocking lever moves between well-oiled guides which prevent any lateral vibration, the whole being firmly bolted to the body of the motor as shown in the photograph of the apparatus (Fig. 2). Linear movements may be obtained by attaching the string to one or other of a number of holes in the rocking-lever. A sliding piece on the crank-shaft (not shown in the figure) which carries a hook and can be fixed at any point enables either circular or elliptical vibrations to be imposed on the extremity of the string if desired.

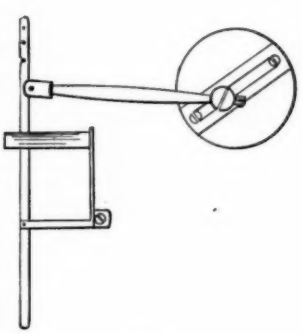


Fig. 1.

The arrangement (modified as described above) has proved to be a most convenient and satisfactory piece of apparatus. It is especially useful in the experimental study or demonstration of the interesting types of vibration maintained by a variable tension, described by the author in previous communications in this REVIEW.[1] These types are obtained when the string is attached to the rocking-lever in such a manner that its oscillation is longitudinal to the string. Under these conditions, as has been shown in the papers quoted, the string may be set in vibration when its frequency in any of its natural modes is sufficiently nearly equal to any multiple of half the frequency of the variation of tension.

[1] PHYS. REV., December, 1912, and July, 1914.

Each of the possible modes has its own characteristic features, and as moreover, the frequency of vibration of the string depends on the number of ventral segments into which it divides up and may thus have simultaneously more than one value, a very large variety of different modes of vibration may be obtained at will by adjusting the tension or length of the string, or by altering the position of the crank-pin of the motor so as to increase or decrease the magnitude of the imposed variation of tension.

The motion of individual points on the string may be very conveniently studied by using a black cord with white dots woven in it as suggested by A. T. Jones and M. E. Phelps in a recent paper in this REVIEW.[1] The form of the figures thus observed reveals at a glance not only the relation of frequency between the longitudinal and transverse movements of any point on the string, but also their phase-relation which appreciably varies with the experimental conditions. Much longer and heavier strings can be used with this apparatus than with an electrically-maintained fork, and by lighting up with the beam from an electric arc, very effective demonstrations are possible. To illustrate the character of the phenomena observed, photographs of the motion of individual points on the string in the first five types of vibration have been secured and are reproduced in Figs. 3 to 12. Figs. 3 and 4 show the first type of vibration in which the ratio of frequencies is 1 : 2, the phase-relation being slightly different in the two cases. In Figs. 5 and 6 the frequency relation is 2 : 2, the curve in Fig. 6 being a deformed ellipse. In Figs. 7, 8 and 9 we have the third type in which the frequency ratio is 3 : 2, the difference in phase being specially marked in Fig. 9. The fourth type in which the frequency relation is 4 : 2 is shown in Figs. 10 and 11. The fifth type is shown in Fig. 12, and exhibits the frequency relation 5 : 2.

It may be useful here to consider the explanation of the interesting phenomenon noticed by Jones and Phelps in their paper,[2] that when two or more of the different types are simultaneously maintained, the ratio of the number of ventral segments into which the string respectively divides up often differs from the ratio of the frequencies of maintenance. For instance, it may be found in one case that the string divides up simultaneously into two and five segments either in the same plane or in perpendicular planes, the frequencies being respectively one half and three halves of the frequency of variation of tension. Apparent anomalies of this kind are due to the fact that we are here dealing with *forced* oscillations, and though the string practically vibrates in its normal

[1] PHYS. REV., November, 1917.

[2] *Loc. cit.*

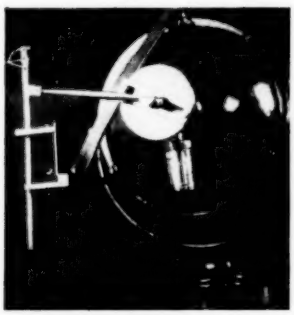
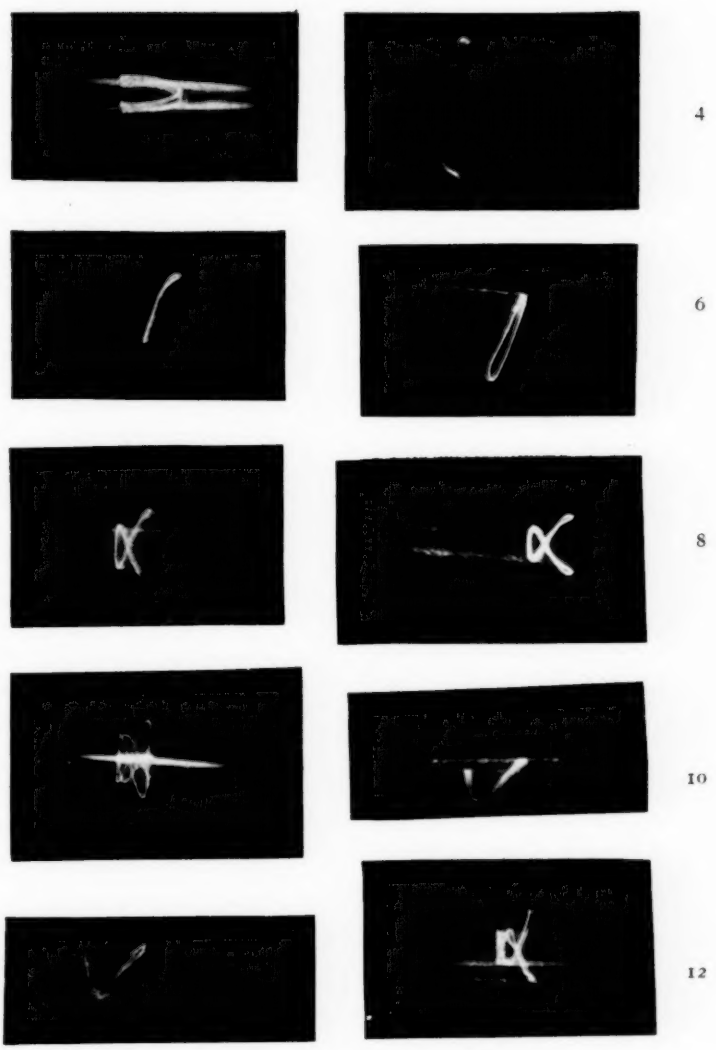


Fig. 2.



Figs. 3–12.

C. V. RAMAN.

modes, the *frequencies* of vibration stand in fixed relations to the impressed forces and may, especially in the case of the first two or three types, differ very appreciably from the natural frequencies of oscillation. Maintenance is, in fact possible when the natural frequency lies anywhere within a certain range which includes the frequency of maintefiance, this range being considerable for the first type, and becoming smaller and smaller as we proceed up the series of types with higher frequencies of maintenance. The magnitudes of the ranges depend on the magnitude of the imposed variations of tension, as may be readily shown with the apparatus described in the present paper, by adjusting the position of the crank-pin, and thus increasing or decreasing the longitudinal motion of the end of the string. With very small variations of tension, the modes with the higher frequencies of maintenance are hardly obtainable, the resonances are much sharper, and the ranges even of the first few types become greatly restricted. With larger variations of tension, the ranges increase, and for the first few types may become very considerable. It will be found that large amplitudes may be obtained even without any exact adjustment for resonance.

The photographs reproduced with the paper were secured for the author by Mr. Rajendra Nath Ghosh, for whose assistance his thanks are due. Mr. Ghosh is now engaged in developing a detailed theory showing the manner in which the amplitudes and phases of the various types of vibration depend on the experimental conditions. The results will, it is hoped, be published in due course.

210, Bowbazar Street,
Calcutta, India,
May 20, 1919.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

A BAROSCOPE FOR MEASURING THE PRESSURE IN EXPLOSION WAVES.

BY DAYTON C. MILLER.

THE baroscope is designed to measure the maximum pressure in an explosive wave, such as that occurring in the air near the muzzle of a large caliber gun, when a shot is fired. The pressure may vary from a few ounces to a thousand pounds per square inch. The conditions of the experiment require that the instrument be rugged and portable.

A diaphragm of elastic material is very tightly clamped between heavy bronze rings, Fig. 1, leaving the central portion free to vibrate. The diaphragm is

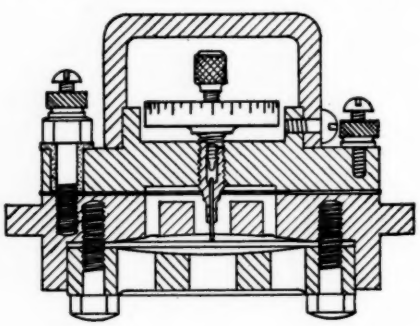


Fig. 1.

open to the air on one side, while the other side is entirely enclosed by a rigid air-tight housing.

The housing carries an insulated micrometer screw; the end of the screw is split and holds by very light friction a sliding contact pin; this pin consists of a fine sewing needle, the gold-plated end of which is placed in contact with the center of the diaphragm. A protecting cover is provided for the graduated head of the screw.

When the diaphragm is caused to vibrate, as by the pressure wave from the explosion, the contact pin is pushed inward by an amount equal to the amplie

[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

tude of the maximum displacement. The amount of this displacement is then determined by connecting the baroscope in an electric circuit with an indicating galvanometer, such as a sensitive voltmeter, and turning the micrometer screw forward till contact with the diaphragm is established.

For resetting the instrument, the micrometer screw is withdrawn to its zero position and the pin is again brought into contact with the diaphragm by means of a small push-screw working in the axis of the micrometer screw. The push-screw is finally withdrawn a turn or two to give opportunity for the pin to move inward at the next pressure measurement.

The baroscope is calibrated in terms of static pressure by means of a hydraulic press provided with a standardized gauge. The baroscope is clamped in a fitting on the upper end of the hydraulic cylinder, there being an air cushion between the water and the diaphragm. Each instrument is then calibrated when it is in the same condition as in the field tests.

For the rapid plotting of a field of pressure, it is desirable to have a number of baroscopes to determine the pressure at various points simultaneously. One instrument can be used in a fixed position for successive experiments, thus, tying together the various sets of readings. Two or more baroscopes may be placed in one position to give an average value as far as instrumental variations are concerned.

For pressures from a few ounces to a thousand pounds per square inch, the diaphragm may be of hard-rolled, spring brass, 75 mm. in outside diameter, having an effective diameter inside the clamping ring of 50 mm. Four different thicknesses have been employed, from 0.38 mm. to 1.66 mm. thick. These have free periods varying from about 1,000 to 5,000. A static pressure of 1 kilogram per square centimeter produces a displacement of the center of the thinnest diaphragm of 0.57 mm., and of the thickest one, 0.016 mm.

The baroscopes described may be used within a few feet of the muzzles of the largest guns, where a less rugged apparatus would be destroyed. For locations where it is safe for a person to remain, a baroscope on the same principle, but using a glass diaphragm, has been used. In this case the movements of the diaphragm are recorded on a moving photographic film by means of the author's phonodeik, and a complete record of the wave form, together with a time scale from a tuning fork interrupter, is obtained. This apparatus is also calibrated in terms of static pressure by means of an air pump and a mercury gauge. With a glass diaphragm of 30 mm. free diameter and 0.085 mm. thick, a change of pressure of 0.16 gram per square centimeter gives a displacement of the light spot on the photographic film of 1 centimeter.

The results of investigations made with the baroscope will be given in full in another paper.

CASE SCHOOL OF APPLIED SCIENCE,
CLEVELAND, OHIO,
April 25, 1919.

HIGH FREQUENCY CURRENTS ON WIRES.[1]

BY J. O. MAUBORGNE.

A PAPER describing original experiments in multiplex telegraphy and telephony was published in 1911 by Major (now Major General) George O. Squier. Previous to this, and even for some time afterwards, the opinion prevailed that the use of high frequency currents as "carriers" for transmission of energy over wire and cable circuits of appreciable length was impossible because of the excessive attenuation which would occur. In fact, even up to the present date consensus of opinion in the scientific world has been that it would be impracticable to use currents of frequencies greater than thirty thousand cycles. The experiments of 1910–1911 made by General Squier disproved the idea that it was not possible to use high-frequency currents under these circumstances and that simultaneous operation of the "wire" and high-frequency systems could be carried on without causing interference to each other. It was further demonstrated that several high-frequency units, tuned to different frequenices, could operate either simultaneously or independently on the same wire line. The frequencies utilized by General Squier at that time did not exceed one hundred thousand cycles, due only to the form of generator used.

Recently this subject has assumed an important aspect from a military standpoint and it was decided to conduct further experiments with the object of determining whether or not it was possible to adapt certain existing types of radiotelephone and telegraph apparatus to multiplex operation. In Signal Corps radio apparatus of the continuous wave type, the three electrode vacuum tube is employed as a generator; the powers dealt with are small (in the neighborhood of six watts) and the frequencies are of the order of 500,000 cycles per second, which is much higher than have hitherto been used for multiplex transmission; such frequencies, up to the present time, have been considered entirely impossible for use on account of the computed excessive attenuation mentioned above. The particular type of apparatus employed in these tests is known as the SCR-67, which is the ground set of the standard ground-airplane radiotelephone equipment.

To provide a practical means for carrying out of the tests, a wire running from Washington to Baltimore, Md., was furnished by the Postal Telegraph Company. One multiplex station was established at the Signal Corps Radio Laboratory, Bureau of Standards, Washington, and a corresponding station at Dixon's Park, Curtis Bay, Md., approximately three miles from the Postal office in Baltimore. The total wire distance between the two stations was sixty miles.

Satisfactory two-way communication was obtained. Speech was received at both stations, loud and with exceptional clearness, the distortion, common

[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

to long-distance wire telephony, being completely absent. The tuning at the receiver was definite, comparable in every respect to that when receiving signals from a "sharply" tuned radio station. This fact will permit of the operation of a number of multiplex units, each tuned to a different frequency on the same line without the use of filter circuits. The carrier frequency employed in these tests was 600,000 cycles per second (wave-length 500 meters); the effective line current, measured at each of the transmitting stations, averaged 60 milamperes.

The range of the SCR-67 when operating as a ground "radio" set in communication with a corresponding SCR-67 is, under ordinary conditions, 10 miles. Thus by confining and directing the flow of energy along a wire the range of communication is materially increased.

OFFICE OF THE CHIEF ENGINEER,
ENGINEERING AND RESEARCH DIVISION, SIGNAL CORPS,
WASHINGTON, D. C., April 25, 1919.

THE PRODUCTION OF VACUUM TUBES FOR MILITARY PURPOSES.[1]

BY N. H. SLAUGHTER.

AT the outset of this country's participation in the European Was, it became evident that there would be required for the signalling devices needed by the Army, enormous quantities of vacuum tubes. (I am assuming that the use of the term "vacuum tube" to designate the three-electrode thermionic device originally called the "audion" is sufficiently understood to obviate the necessity of any further explanation.) The problem was not essentially a "research" problem, as the term is usually understood, but was rather a problem of design and production of vacuum tubes already partly developed in research laboratories.

Previous State of the Art.—In April, 1917, the state of the vacuum tube art was more or less as follows:

In the United States the development of vacuum tubes had progressed to the point where these tubes were in considerable use as detectors and amplifiers of radio signals for amateurs and to a limited extent for government radio stations and for use as amplifiers or "telephone repeaters" by the American Telephone and Telegraph Company. In addition, transmitting tubes had been developed sufficiently to use in experimental radio telephone sets. These vacuum tubes, however, were not a standardized product, were difficult to manufacture, and varied widely in their individual characteristics.

In France the stimulus of war had accelerated the development of vacuum tubes so that there was in production and use by the French signalling units, tubes of a comparatively satisfactory type. The French had adopted the practice of employing only a single type of tube for all purposes. However,

[1] Abstract of a paper presented at the Washington meeting of the American Physical Society, April 25 and 26, 1919.

the necessity for adopting a tube which was a compromise in so far as the various operating characteristics are concerned and a further compromise from the standpoint of ease and cheapness of manufacture, had so handicapped the design of the tube that the resultant product was far from ideal in its performance of any of the particular duties imposed upon it. In Great Britain the vacuum tube art had progressed along much the same lines as in France, but the military forces had not adopted this device to the same extent as the French.

Basic Specifications.—In order to arrive at the basic specifications for vacuum tubes for the United States military forces, careful study was made of the characteristics of the French tubes, with the result that the specifications for the American vacuum tubes show marked resemblance in some respects to the French tubes. The fundamental basis of the specifications for any military device comprise ruggedness, reliability, minimum of adjustment and for portable equipment the irreducible minimum of power consumption. In addition to these requirements, it was necessary to standardize the filament voltage and current input, the plate voltage and current input and numerous other detailed points involved in the design of the vacuum tubes. Because of the great difference in the operating requirements for transmitting and receiving tubes, it was decided to adopt two types, one for transmitting purposes and the other for receiving purposes. For the receiving tube the filament voltage was chosen at a value which would conform with the French, namely, the voltage obtained from a two-cell lead storage battery, while the filament current was specified as the minimum which could be obtained and allow the tube to meet other requirements. The plate voltage of the receiving tube was chosen as the voltage given by a 15-cell dry battery, this value being arrived at as the minimum which would insure proper operation of the receiving tubes and yet not require an excessively heavy plate battery. The value of plate current was correspondingly limited to less than 1 mil in order to make it possible to utilize a small size of dry cell for supplying this current. Fortunately the above specifications were substantially met by a design of tube already partially standardized by a commercial company. The most noteworthy improvements that have been effected in the receiving tube during the process of development and manufacture of this tube for the Signal Corps, have been a new type of structure for supporting the electrodes, which new type is enormously more rugged than the type existing in April, 1917; and a reduction of filament current to less than 20 per cent. of the original value. The development of the tube in other respects has been largely in the nature of minor improvements to secure better uniformity and a tube better adapted to quantity production methods. The use of receiving tubes requires satisfactory performance as detectors of radio frequency signals and amplifiers of radio frequency or radio frequency signals, together with the ability to operate as an oscillating detector for heterodyne reception.

The basis for the electrical specifications for transmitting tubes was an output

of 5 watts when operating as a high-frequency oscillator. Other specifications called for 300 volts plate potential and values of filament voltage and current of 8 volts and 1.35 amperes. The latter values were arrived at after a considerable period of experimentation to determine the best possible values which could be assigned for these quantities.

The operation of the transmitting tube required performance as oscillators for the generation of high-frequency current for transmitting and as modulators for the control of this high-frequency current for radio telephony.

Production.—Prior to July, 1917, the production of vacuum tubes in the United States had never exceeded more than a few hundred tubes per week. This production had been accomplished on what was practically a laboratory hand-made basis. The problem of producing vacuum tubes at rates of several thousands per day was truly a stupendous one when viewed from the standpoint of the production situation at that time. The design of the two types of tubes required by the Signal Corps was put into the hands of the engineers of the three leading vacuum tube manufacturers of the country and sufficient initial production was authorized to enable the process of manufacture of the tubes to be worked out as the designs were perfected. The final specifications represent a compromise between the characteristics desired from an operational standpoint and the characteristics adapted to quantity production methods. In November, 1918, deliveries of Standard Signal Corps vacuum tubes were being made at a rate of more than one hundred thousand tubes per month.

The difficulties encountered by the manufacturers in undertaking the production of vacuum tubes were largely labor difficulties, on account of the fact there existed no personnel trained in the processes required for the quantity production of these tubes. The equipment required for machine production of the various parts of the tube, the pumps required for evacuation, and the extensive testing facilities required for insuring that the product came up to the standards prescribed, combined to make the production of vacuum tubes a problem of extreme difficulty. The greatest credit is due to the engineers of the commercial companies engaged in the production of these tubes for the ingenuity and enthusiasm with which they solved the problems incident to this work.

A glance at the specifications for acceptance of vacuum tubes would indicate a large number of tests to which a given vacuum tube is subjected before being considered an acceptable tube, but the development of suitable testing equipment and methods has resulted in the simplification of this problem so that the government inspector is enabled to inspect several thousand tubes per day without difficulty.

Performance.—The performance of the standard Signal Corps vacuum tubes in thousands of airplane and land radio sets is conclusive proof of the satisfactory character of these tubes for military purposes. As an example, it may be stated that data collected from many thousands of flights made by airplanes using the radio telephone set for voice-commanded squadron drills, indicates

that there is much less trouble from the complete radio equipment than is experienced from the extremely efficient airplane engines. When we consider that the complete radio sets contain many other sources of trouble in addition to vacuum tubes, it will be realized that this is a remarkable tribute to the designers and manufacturers of the vacuum tube.

While predictions as to the future military policy of the United States are not in order at the present time, it can safely be said that the vacuum tube has taken its place as an essential part of military equipment and that the progress of its future development will be a matter of the greatest military importance to the country. Even more important, however, is the influence which the war-time development of this device will exert on the scientific and commercial progress of electrical engineering in the immediate future.

NEW BOOKS.

A Handbook of Physics Measurements. By E. S. FERRY in Collaboration with O. W. SILVEY, G. W. SHERMAN, JR., and D. C. DUNCAN. Two volumes. New York, John Wiley and Sons, Inc., 1918. Pp. ix + 251 and x + 233.

These two volumes contain the description of over one hundred experiments, affording ample selection for a laboratory course in general physics such as is given to engineering students. The authors have aimed "to furnish the student . . . with a self-contained manual of the theory and manipulation of those measurements in physics which bear most directly upon his subsequent work in other departments of study and upon his future professional career." For example, experiments in harmonic oscillations, both damped and undamped, suitable for electrical engineers, others such as combustion calorimeters and study of efficiency of pipe coverings, suitable for mechanical engineers, are given.

The experiments of each chapter are preceeded by rather full theoretical treatment which the student is expected to digest before he begins his experimental work. In their own laboratory, the authors expect the student to bring to the laboratory for the inspection of the instructor, a written analysis of the experiment with a discussion of the theory involved, before he starts the experimental work.

The first volume contains experiments in mechanics, the properties of matter, and optics; the second has experiments in vibratory motion, sound, heat, electricity, and magnetism.

In the second volume, calculus methods are used whenever in the opinion of the authors, its use would result in economy of time and effort, but even then only in a relatively small number of experiments.

The general plan of the books as well as the detailed working out of each topic shows that the authors have put a great deal of thought and effort into these manuals. O. M. S.

Sound, Light, Electricity and Magnetism. By W. B. ANDERSON. New York, McGraw-Hill Book Co., Inc., 1919. Pp. xiii + 794.

This volume is of similar scope and treatment to that on "Mechanics and Heat" by the same author, and is intended as part of a first course in college physics. The book is written primarily for engineering and agricultural students; the practical side of the subject is emphasized and there is included a well selected list of problems. Both author and publisher have done a most excellent piece of work. The text is very clear, paragraphs are numbered and headed and there are some 373 well-made illustrations.

The subject matter is perhaps somewhat more voluminous than is usually offered in a first college course, although there is provision for a shorter course by omitting judiciously chosen sections in finer print. The calculus is not used and the question might perhaps be raised whether it is not better to defer treatment of some of the more complex subjects and their applications until the student has acquired at least an elementary knowledge of this branch of mathematics.

G. K. B.